International Journal for Research in Applied Sciences and Biotechnology

ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

www.ijrasb.com

https://doi.org/10.31033/ijrasb.9.1.3

Unified Finite Integral and Modified of Generalized Aleph-Function of Two Variables

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ABSTRACT

In the present paper, we evaluate the general finite integral invoving the generalized modified Aleph-function of two variables. At the end, we shall see several corollaries and remarks.

Keywords- Generalized modified Aleph-function of two variables, generalized modified I-function of two variables, generalized modified H-function of two variables, generalized modified Meijer-function of two variables, Aleph-function of two variables, I-function of two variables, H-function of two variables, Meijer G-function of two variables, Two Mellin-Barnes integrals contour, elliptic integrals of first species.

2010 Mathematics Subject Classification: 33C05, 33C60.

I. INTRODUCTION

Recently Aleph-function of two variables has been introduced and studied by Sharma [14], Kumar [7], it's an extension of I-function of two variables defined Sharma and Mishra [15] which is a generalization of the H-function of two variables due to Gupta and Mittal. [6]. On the other hand, Prasad and Prasad [10] have defined the modified generalized H function of two variables. In this paper, first, we introduce and study the generalized modified Aleph-function of two variables. This function unifies the Aleph-function of two variables and the modified of generalized H-function of two variables. Later, we calculate a finite generalized integral involving this function. At the end, we will give several spe - cial cases and remarks.

The double Mellin-Barnes integrals contour occurring in this paper will be referred to as the generalized function of two variables throughout our present study and will be defined and represented as follows:

$$\aleph_1(z_1,z_2) = \aleph^{m_1,n_1;m_2,n_2:m_3,n_3;m_4,n_4}_{P_i,Q_i,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|l} \mathrm{z}_1 & (\mathrm{a}_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} : \\ \cdot & \qquad\cdot \\ \cdot & \qquad\cdot \\ \mathrm{z}_2 & (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i} : \end{array}\right.$$

$$\left.\begin{array}{l} (\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}} : (\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \qquad\cdot \\ \qquad\cdot \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}} :(\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)x^s y^t \mathrm{d}s\mathrm{d}t, \tag{1.1}$$

where

$$\phi(s,t) = \frac{\prod_{j=1}^{n_1}\Gamma(1-a_j+\alpha_j s+A_j t)\prod_{j=1}^{m_1}\Gamma(b_j-\beta_j s-B_j t)}{\sum_{i=1}^{r}\tau_i\left[\prod_{j=m_1+1}^{Q_i}\Gamma(1-b_{ji}+\beta_{ji}s+B_{ji}t)\prod_{n_1+1}^{P_i}\Gamma(a_{ji}-\alpha_{ji}s-A_{ji}t)\right]}$$

$$\frac{\prod_{j=1}^{n_2}\Gamma(1-a'_j+\alpha'_j s-A'_j t)\prod_{j=1}^{m_2}\Gamma(b'_j-\beta'_j s+B'_j t)}{\sum_{i'=1}^{r'}\tau_{i'}\left[\prod_{j=m_2+1}^{Q_{i'}}\Gamma(1-b'_{ji'}+\beta'_{ji'}s-B'_{ji'}t)\prod_{n_2+1}^{P_{i'}}\Gamma(a'_{ji'}-\alpha'_{ji'}s+A'_{ji'}t)\right]} \tag{1.2}$$

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.3

$$\theta_1(s)=\frac{\prod_{j=1}^{m_3}\Gamma(d_j-\delta_j s)\prod_{j=1}^{n_3}\Gamma(1-c_j+\gamma_j s)}{\sum_{i''=1}^{r''}\tau_{i''}\left[\prod_{j=m_3+1}^{Q_{i''}}\Gamma(1-d_{ji''}+\delta_{ji''}s)\prod_{j=n_3+1}^{P_{i''}}\Gamma(c_{ji''}-\gamma_{ji''}s)\right]} \tag{1.3}$$

$$\theta_2(t)=\frac{\prod_{j=1}^{m_4}\Gamma(f_j-F_j t)\prod_{j=1}^{n_4}\Gamma(1-e_j+E_j t)}{\sum_{i'''=1}^{r'''}\tau_{i'''}\left[\prod_{j=m_4+1}^{Q_{i'''}}\Gamma(1-f_{ji'''}+F_{ji'''}t)\prod_{j=n_4+1}^{P_{i'''}}\Gamma(e_{ji'''}-E_{ji'''}t)\right]} \tag{1.4}$$

where z_1 and z_2 (real or complex) are not equal to zero and an empty product is interpreted as unity and the quantities

$P_i, P_{i'}, P_{i''}, P_{i'''}, Q_i, Q_{i'}, Q_{i''}, Q_{i'''}, m_1, m_2, m_3, m_4, n_1, n_2, n_3, n_4$ are non-negative integers such that

$Q_i>0, Q_{i'}>0, Q_{i''}>0, Q_{i'''}>0; \tau_i, \tau_{i'}, \tau_{i''}, \tau_{i'''}>0 (i=1,\cdots,r), (i'=1,\cdots,r'), (i''=1,\cdots,r''), (i'''=1,\cdots,r''')$.

All the $A's, \alpha's, B's, \beta's, A''s, B''s, \alpha''s, \beta''s, \gamma's, \delta's, E's$ and $F's$ are assumed to be positive quantities for standardization purpose ; the definition of generalized Aleph-function of two variables given above will however, have a meaning even if some of these quantities are zero. The definition of generalized Aleph-function of two variables given above will however, have a meaning even if some of these quantities are zero and the numbers $a_j, b_j, a'_j, b'_j, b'_{ji'}, a'_{ji'}, b_{ji'}, a_{ji'}$, c_j, d_j, $d_{ji''}, c_{ji''}$, $f_i, g_i, f_{ji'}, g_{ji''}$ are complex numbers. The contour L_1 is in the s-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(b_j-\beta_j s-B_j t)(j=1,\cdots,m_1)$, $\Gamma(d_j-\delta_j s)(j=1,\cdots,m_3)$ and $\Gamma(b'_j-\beta'_j s+B'_j t)(j=1,\cdots,m_2)$, are to the right and all the poles of $\Gamma(1-a_j+\alpha_j s+A_j t)(j=1,\cdots,n_1)$, $\Gamma(1-c_j+\gamma s)$,$(j=1,\cdots,n_3)$ and $\Gamma(1-a'_j+\alpha'_j s-A'_j t)(j=1,\cdots,n_2)$ lie to the left of L_1. The contour L_2 is in the t-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(b_j-\beta_j s-B_j t)(j=1,\cdots,m_1)$, $\Gamma(f_j-F_j t)(j=1,\cdots,m_4)$ $\Gamma(1-a'_j+\alpha'_j s-A'_j t)(j=1,\cdots,n_2)$are to the right and all the poles of $\Gamma(1-a_j+\alpha_j s+A_j t)(j=1,\cdots,n_1)$, $\Gamma(1-e_j+E_j t)(j=1,\cdots,n_3)$ and$\Gamma(b'_j-\beta'_j s+B'_j t)(j=1,\cdots,m_2)$ lie to the left of L_2. The poles of the integrand are assumed to be simple.

The function defined by the equation (3.1) is analytic function of z_1 if and z_2 if

$$U_1=\tau_i\sum_{j=1}^{P_i}\alpha_{ji}-\tau_i\sum_{j=1}^{Q_i}\beta_{ji}+\tau_{i'}\sum_{j=1}^{P_{i'}}\alpha_{ji'}-\tau_{i'}\sum_{j=1}^{Q_{i'}}\beta_{ji'}+\tau_{i''}\sum_{j=1}^{P_{i''}}\gamma_{ji''}-\tau_{i''}\sum_{j=1}^{Q_{i''}}\delta_{ji''}<0 \tag{1.5}$$

$$U_2=\tau_i\sum_{j=1}^{P_i}A_{ji}-\tau_i\sum_{j=1}^{Q_i}B_{ji}+\tau_{i'}\sum_{j=1}^{P_{i'}}A_{ji'}-\tau_{i'}\sum_{j=1}^{Q_{i'}}B'_{ji'}+\tau_{i'''}\sum_{j=1}^{P_{i'''}}E_{ji'''}-\tau_{i'''}\sum_{j=1}^{Q_{i'''}}F_{ji'''}<0 \tag{1.6}$$

The integral defined by (2.1) is converges absolutely, if

$$U_3=\sum_{j=1}^{n_1}\alpha_j+\sum_{j=1}^{n_2}\alpha'_j+\sum_{j=1}^{m_1}\beta_j+\sum_{j=1}^{m_2}\beta'_j+\sum_{j=1}^{n_3}\gamma_j+\sum_{j=1}^{m_3}\delta_j-\tau_i\max_{1\le i\le 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji}+\sum_{j=m_1+1}^{Q_i}\beta_{ji}\right)$$

$$-\tau'_i\max_{1\le i'\le 3}\left(\sum_{j=n_2+1}^{P'_i}\alpha_{ji'}+\sum_{j=m_2+1}^{Qi'}\beta_{ji'}\right)-\tau''_i\max_{1\le i''\le 3}\left(\sum_{j=n_3+1}^{P''_i}\alpha_{ji''}+\sum_{j=m_3+1}^{Q''_i}\beta_{ji''}\right)>0 \tag{1.7}$$

and

$$U_4=\sum_{j=1}^{n_1}\alpha_j+\sum_{j=1}^{n_2}\alpha'_j+\sum_{j=1}^{m_1}\beta_j+\sum_{j=1}^{m_2}\beta'_j+\sum_{i=1}^{m_4}E_j+\sum_{j=1}^{n_4}F_j-\tau_i\max_{1\le i\le 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji}+\sum_{j=m_1+1}^{Q_i}\beta_{ji}\right)$$

$$-\tau'_i\max_{1\le i'\le 3}\left(\sum_{j=n_2+1}^{P'_i}\alpha_{ji'}+\sum_{j=m_2+1}^{Qi'}\beta_{ji'}\right)-\tau'''_i\max_{1\le i'''\le 3}\left(\sum_{j=n_4+1}^{P'''_i}E_{ji'''}+\sum_{j=m_4+1}^{Q'''_i}F_{ji'''}\right)>0 \tag{1.8}$$

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.3

$|argz_1| < \frac{\pi}{2}U_3$ and $|argz_2| < \frac{\pi}{2}U_4$

We may establish the the asymptotic behavior in the following convenient form, see B.L.J. Braaksma [5]:

$$\aleph(z_1,z_2) = 0(\ |z_1|^{\alpha_1},\ |z_r|^{\alpha_2}\ ),\ max(\ |z_1|,\ |z_2|\ ) \to 0$$

$$\aleph(z_1,z_2) = 0(\ |z_1|^{\beta_1},|z_2|^{\beta_2}\ ),\ min(\ |z_1|,|z_2\ |\ ) \to \infty\ :$$

where $\alpha_1 = \min_{1\leqslant j\leqslant m_3} Re\left[\left(\frac{d_j}{\delta_j}\right)\right]$ and $\alpha_2 = \min_{1\leqslant j\leqslant m_4} Re\left[\left(\frac{f_j}{F_j}\right)\right]$

where $\beta_1 = \max_{1\leqslant j\leqslant n_3} Re\left[\left(\frac{c_j-1}{\gamma_j}\right)\right]$ and $\beta_2 = \max_{1\leqslant j\leqslant n_4} Re\left[\left(\frac{e_j-1}{E_j}\right)\right]$

II. REQUIRED INTEGRAL

In this section, we give an integral evaluated by Prudnikov et al. ([11], 2.2.9 Eq3 page 309). In the following, it should also be noted that for integrals there exist several different representations which are written as distinct formulas but without repetition of the left-hand side.

Lemma

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}\Gamma(\alpha-\sigma)}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}\Gamma\left(\alpha+\frac{1}{2}-\sigma\right)}\left\{\begin{matrix} \mathrm{c}^{-\frac{1}{2}} \\ (\mathrm{a+2b+c})^{-\frac{1}{2}} \end{matrix}\right\} \quad (2.1)$$

where $\mathrm{Re}(\alpha) > 0, \sigma = \left\{\begin{matrix} 0 \\ 1 \end{matrix}\right\}$, $\beta = \left\{\begin{matrix} \alpha \\ \alpha-2 \end{matrix}\right\}$ and c, $a+2b+c>0,\ a<(\sqrt{c}+\sqrt{a+2b+c})^2$

III. MAIN INTEGRAL

In this section, we study a generalization of the finite integral involving the modified of generalized Aleph-function of two variables. We have the unified integral.

Theorem

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x\ \aleph\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$\left\{\begin{matrix} \mathrm{c}^{-\frac{1}{2}} \\ (\mathrm{a+2b+c})^{-\frac{1}{2}} \end{matrix}\right\} \aleph^{m_1,n_1+1;m_2,n_2:m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}$$

$$\left(\begin{matrix} \frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^A}\ \mathrm{z}_1 \\ \cdot \\ \cdot \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^B}\ \mathrm{z}_2 \end{matrix}\ \middle|\ \begin{matrix} (1+\sigma-\alpha;A,B),(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot \\ \cdot \\ (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},(\frac{1}{2}+\sigma-\alpha;A,B): \end{matrix}\right.$$

$$\left.\begin{matrix} (\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}:(\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{matrix}\right) \quad (3.1)$$

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Provided

$$0 < Re(\alpha) + A \min_{1\leqslant j\leqslant m_3} Re\left(\frac{d_j}{\delta_j}\right),\ 0 < Re(\beta) + B \min_{1\leqslant j\leqslant m_4} Re\left(\frac{f_j}{F_j}\right)$$

$A, B > 0$, $|argz_1| < \frac{1}{2}U_3\pi$, $|argz_2| < \frac{1}{2}U_4\pi$, U_3 and U_4 are defined respectively by the equations (1.7) and (1.8).

Proof.

To prove the theorem, expressing the modified generalized Aleph-function of two variables in double Mellin-Barnes contour integrals with the help of (1.1) and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collecting the power of x, we obtain J

$$\mathrm{J} = \int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x\ \aleph\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x =$$

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^{As}\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^{Bt}\mathrm{dsdt\ dx} \quad (3.2)$$

We can write

$$\mathrm{J} = \int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x\ \aleph\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x =$$

$$\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^{As}\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^{Bt}\mathrm{dx\ dsdt} \quad (3.3)$$

and modified the above expression, this give :

$$\mathrm{J} = \int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x\ \aleph\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x =$$

$$\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t\int_0^1 \frac{x^{\alpha+As+Bt-1}(1-x)^{\beta+As+Bt}}{(ax^2+2bx+c)^{\alpha+As+Bt+\frac{1}{2}-\sigma}}\mathrm{d}x\mathrm{d}s\mathrm{d}t \quad (3.4)$$

We use the lemma, this gives:

$$\mathrm{J} = \int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x\ \aleph\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \left\{\begin{matrix} \mathrm{c}^{-\frac{1}{2}} \\ (\mathrm{a}+2\mathrm{b}+\mathrm{c})^{-\frac{1}{2}} \end{matrix}\right\}$$

$$\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t\frac{2^{-\alpha-As-Bt+\sigma}\sqrt{\pi}\Gamma(\alpha+As+Bt-\sigma)}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha+As+Bt-\sigma}\Gamma\left(\alpha+As+Bt+\frac{1}{2}-\sigma\right)}\mathrm{dsdt} \quad (3.5)$$

where $\sigma = \left\{\begin{matrix} 0 \\ 1 \end{matrix}\right\}$ and $\beta = \left\{\begin{matrix} \alpha \\ \alpha-2 \end{matrix}\right\}$, By helping the definition of the modified of generalized Aleph-function of two variables, after a few simplifications, we get the relation (3.1). Now, we interest to particular cases.

IV. SPECIAL CASES

Let the generalized modified Aleph-function of two variables reduces to modified of generalized I-function of two variables, this gives

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.3

Corollary 1

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x\ \mathrm{I}\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$\left\{\begin{matrix} \mathrm{c}^{-\frac{1}{2}} \\ (\mathrm{a+2b+c})^{-\frac{1}{2}} \end{matrix}\right\} I^{m_1,n_1+1;m_2,n_2:m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1;r;P_{i'},Q_{i'};r';P_{i''},Q_{i''};r'':P_{i'''},Q_{i'''};r'''}$$

$$\left(\begin{matrix} \frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^A}\,\mathrm{z}_1 \\ \cdot \\ \cdot \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^B}\,\mathrm{z}_2 \end{matrix}\ \middle|\ \begin{matrix} (1+\sigma-\alpha;A,B),(a_j,\alpha_j,A_j)_{1,n_1},[(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot \\ \cdot \\ (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},(\frac{1}{2}+\sigma-\alpha;A,B): \end{matrix}\right.$$

$$\left.\begin{matrix} (\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (\mathrm{c}_j,\gamma_j)_{1n_3},[(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}\ (\mathrm{d}_j,\delta_j)_{1m_3},[(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{matrix}\right) \quad (4.1)$$

under the same notations and conditions that the fundamental theorem with $\tau_i,\tau_{i'},\tau_{i''},\tau_{i'''}\to 1$, $|argz_1| < \frac{1}{2}U'_3\pi$, $|argz_2| < \frac{1}{2}U'_4\pi$, U'_3 and U'_4 are defined by

$$U_3 = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_2}\alpha'_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{m_2}\beta'_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \max_{1\le i\le 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji} + \sum_{j=m_1+1}^{Q_i}\beta_{ji}\right)$$

$$-\max_{1\le i'\le 3}\left(\sum_{j=n_2+1}^{P'_i}\alpha_{ji'} + \sum_{j=m_2+1}^{Qi'}\beta_{ji'}\right) - \max_{1\le i''\le 3}\left(\sum_{j=n_3+1}^{P''_i}\alpha_{ji''} + \sum_{j=m_3+1}^{Q''_i}\beta_{ji''}\right) > 0 \quad (4.2)$$

and

$$U_4 = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_2}\alpha'_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{m_2}\beta'_j + \sum_{i=1}^{m_4}E_j + \sum_{j=1}^{n_4}F_j - \max_{1\le i\le 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji} + \sum_{j=m_1+1}^{Q_i}\beta_{ji}\right)$$

$$-\max_{1\le i'\le 3}\left(\sum_{j=n_2+1}^{P'_i}\alpha_{ji'} + \sum_{j=m_2+1}^{Qi'}\beta_{ji'}\right) - \max_{1\le i'''\le 3}\left(\sum_{j=n_4+1}^{P'''_i}E_{ji'''} + \sum_{j=m_4+1}^{Q'''_i}F_{ji'''}\right) > 0 \quad (4.3)$$

We consider the above corollary and we suppose r = r' = r" = r"' =1, we obtain the generalized modified H-function defined by Prasad and Prasad [10] and the following integral.

Corollary 2

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x\ \mathrm{H}\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.3

$$\left\{\begin{matrix} c^{-\frac{1}{2}} \\ (a+2b+c)^{-\frac{1}{2}} \end{matrix}\right\} H^{m_1,n_1+1;m_2,n_2:m_3,n_3;m_4,n_4}_{P+1,Q+1:P',Q':P'',Q'':P''',Q'''}\left(\begin{matrix} \frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^A}\, z_1 \\ \cdot \\ \cdot \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^B}\, z_2 \end{matrix}\left|\begin{matrix} (1+\sigma-\alpha;A,B),(a_j,\alpha_j,A_j)_{1,p_1}: \\ \cdot \\ \cdot \\ (b_j,\beta_j,B_j)_{1,q_1},(\frac{1}{2}+\sigma-\alpha;A,B): \end{matrix}\right.\right.$$

$$\left.\begin{matrix} (a'_j,\alpha'_j,A_j)_{1,P_2}:(c_j,\gamma_j)_{1,P_3};(e_j,E_j)_{1,P_4} \\ \cdot \qquad\qquad \cdot \\ \cdot \qquad\qquad \cdot \\ (b'_j,\beta'_j,B_j)_{1,Q_2}:(d_j,\delta_j)_{1,Q_3};(f_j,F_j)_{1,Q_4} \end{matrix}\right) \tag{4.4}$$

with the same notations and conditions that corollary 1 and $r = r' = r'' = r''' = 1$, $|argz_1| < \frac{1}{2}U''_3\pi$, $|argz_2| < \frac{1}{2}U''_4\pi$, U''_3 and U''_4 are defined by :

$$U''_3 = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_2}\alpha'_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{m_2}\beta'_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \sum_{j=n_1+1}^{P_1}\alpha_j - \sum_{j=m_1+1}^{Q_1}\beta_j - \sum_{j=m_2+1}^{Q_2}\beta_j$$

$$\sum_{j=n_2+1}^{P_2}\alpha_j - \sum_{j=m_3+1}^{Q_3}\delta_j - \sum_{j=n_3+1}^{P_3}\gamma_j > 0 \tag{4.5}$$

$$U''_4 = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{n_2}A'_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{m_2}B'_j + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \sum_{j=n_1+1}^{P_1}A_j - \sum_{j=m_1+1}^{Q_1}B_j - \sum_{j=m_2+1}^{Q_2}B_j$$

$$-\sum_{j=n_2+1}^{P_2}A_j - \sum_{j=m_4+1}^{Q_4}F_j - \sum_{j=n_4+1}^{P_4}E_j > 0 \tag{4.6}$$

Taking $(\alpha_j)_{1,P_1} = (A_j)_{1,P_1} = (\alpha'_j)_{1,P_2} = (A'_j)_{1,P_2} = (\gamma_j)_{1,P_3} = (E_j)_{1,P_4} = (\beta_j)_{1,Q_1} = (B_j)_{1,q_1} = (\beta'_j)_{1,Q_2} = 1 =$ $= (B'_j)_{1,Q_2} = (\delta_j)_{1,Q_3} = (F_j)_{1,Q_4} = A = B$, the modified generalized H-function of two variables is replaced by the generalized modified of Meijer G-function of two variables defined by Agarwal [1], we have :

Corollary 3

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}dx\ \ G\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right), z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)\right]dx = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{[b+c+\sqrt{c}\sqrt{a+2b+c}]^{\alpha-\sigma}}$$

$$G^{m_1,n_1+1;m_2,n_2:m_3,n_3;m_4,n_4}_{P+1,Q+1:P',Q':P'',Q'':P''',Q'''}\left(\begin{matrix} \frac{2}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]}\, z_1 \\ \cdot \\ \cdot \\ \frac{2}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]}\, z_2 \end{matrix}\left|\begin{matrix} (1+\sigma-\alpha),(a_j)_{1,p_1}: \\ \cdot \\ \cdot \\ (b_j)_{1,q_1},(\frac{1}{2}+\sigma-\alpha): \end{matrix}\right.\right.$$

$$\left.\begin{matrix} (a'_j)_{1,P_2}:(c_j)_{1,P_3};(e_j)_{1,P_4} \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ (b'_j)_{1,Q_2}:(d_j)_{1,Q_3};(f_j)_{1,Q_4} \end{matrix}\right) \tag{4.7}$$

under the conditions verified by the corollary 2 and the conditions mentionned at the beginning of the corollary 3,

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.3

and $|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

$$U_1 = \left[m_1 + n_1 + m_2 + n_2 + m_3 + n_3 - \tfrac{1}{2}(p_1 + q_1 + p_2 + q_2 + p_3 + q_3)\right] \tag{4.8}$$

and

$$V_1 = \left[m_1 + n_1 + m_2 + n_2 + m_4 + n_4 - \tfrac{1}{2}(p_1 + q_1 + p_2 + q_2 + p_4 + q_4)\right] \tag{4.9}$$

Let $m_2 = n_2 = P_{i'} = Q_{i'} = 0$, the modified of generalized Aleph-function of two variables reduces to generalized Aleph-function of two variables, this gives :

Corollary 4

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x \ \aleph\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$\left\{\begin{matrix} c^{-\frac{1}{2}} \\ (a+2b+c)^{-\frac{1}{2}} \end{matrix}\right\} \aleph^{m_1,n_1+1:m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r:P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}$$

$$\left(\begin{matrix} \frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^A}z_1 \\ \cdot \\ \cdot \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^B}z_2 \end{matrix}\left|\begin{matrix} (1+\sigma-\alpha;A,B),(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot \\ \cdot \\ (b_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},(\frac{1}{2}+\sigma-\alpha;A,B): \end{matrix}\right.\right.$$

$$\left.\begin{matrix} (c_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (d_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{matrix}\right) \tag{4.10}$$

Provided the conditions and notations verified by the fundamental theorem with $m_2 = n_2 = P_{i'} = Q_{i'} = 0$, $|argz_1| < \frac{1}{2}U_3'''\pi, |argz_2| < \frac{1}{2}U_4'''\pi$, U_3''' and U_4''' are defined respectively by the relations

$$U_3''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \tau_i \max_{1\le i\le 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji} + \sum_{j=m_1+1}^{Q_i}\beta_{ji}\right)$$

$$-\tau_i'' \max_{1\le i''\le 3}\left(\sum_{j=n_3+1}^{P_i''}\alpha_{ji''} + \sum_{j=m_3+1}^{Q_i''}\beta_{ji''}\right) > 0 \tag{4.11}$$

and

$$U_4''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{i=1}^{m_4}E_j + \sum_{j=1}^{n_4}F_j - \tau_i \max_{1\le i\le 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji} + \sum_{j=m_1+1}^{Q_i}\beta_{ji}\right)$$

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.3

$$-\tau_i''' \max_{1\leq i'''\leq 3}\left(\sum_{j=n_4+1}^{P_i'''} E_{ji'''}+\sum_{j=m_4+1}^{Q_i'''} F_{ji'''}\right)>0 \tag{4.12}$$

Let $\tau_i, \tau_{i''}, \tau_{i'''} \to 1$, the generalized modified Aleph-function of two variables reduces to generalized I-function of two variables, this gives :

Corollary 5

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^\beta}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x \ \mathrm{I}\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$\left\{\begin{matrix} \mathrm{c}^{-\frac{1}{2}} \\ (\mathrm{a+2b+c})^{-\frac{1}{2}}\end{matrix}\right\} I^{m_1,n_1+1:m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1;r:P_{i''},Q_{i''};r'':P_{i'''},Q_{i'''};r'''}$$

$$\left(\begin{matrix}\frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^A}\,\mathrm{z}_1 \\ \cdot \\ \cdot \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^B}\,\mathrm{z}_2\end{matrix}\ \middle|\ \begin{matrix}(1+\sigma-\alpha;A,B),(a_j,\alpha_j,A_j)_{1,n_1},[(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot \\ \cdot \\ (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},(\frac{1}{2}+\sigma-\alpha;A,B):\end{matrix}\right.$$

$$\left.\begin{matrix}(\mathrm{c}_j,\gamma_j)_{1n_3},[(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{d}_j,\delta_j)_{1m_3},[(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}}\end{matrix}\right) \tag{4.13}$$

By using the conditions verified by the above corollary with $\tau_i, \tau_{i''}, \tau_{i'''} \to 1$, $|argz_1| < \frac{1}{2}U_3''\pi, |argz_2| < \frac{1}{2}U_4''\pi$, U_3'' and U_4'' are defined by

$$U_3'' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \max_{1\leq i\leq 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji}+\sum_{j=m_1+1}^{Q_i}\beta_{ji}\right)$$

$$-_{1\leq i''\leq 3}\left(\sum_{j=n_3+1}^{P_i''}\alpha_{ji''}+\sum_{j=m_3+1}^{Q_i''}\beta_{ji''}\right)>0 \tag{4.14}$$

and

$$U_4''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{i=1}^{m_4}E_j + \sum_{j=1}^{n_4}F_j - \max_{1\leq i\leq 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji}+\sum_{j=m_1+1}^{Q_i}\beta_{ji}\right)$$

$$-\max_{1\leq i'''\leq 3}\left(\sum_{j=n_4+1}^{P_i'''}E_{ji'''}+\sum_{j=m_4+1}^{Q_i'''}F_{ji'''}\right)>0 \tag{4.15}$$

We have the generalized H-function of two variables and the result:

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

https://doi.org/10.31033/ijrasb.9.1.3

Corollary 6

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x \;\; \mathrm{H}\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^{A}, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^{B}\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$\left\{\begin{array}{c} \mathrm{c}^{-\frac{1}{2}} \\ (\mathrm{a+2b+c})^{-\frac{1}{2}} \end{array}\right\} H^{m_1,n_1+1:m_3,n_3;m_4,n_4}_{P+1,Q+1:P'',Q'':P''',Q'''}\left(\begin{array}{c|c} \frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^{A}}\, z_1 & (1+\sigma-\alpha;A,B),(a_j,\alpha_j,A_j)_{1,p_1} : (\mathrm{c}_j,\gamma_j)_{1,P_3};(e_j,E_j)_{1,P_4} \\ \vdots & \vdots \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^{B}}\, z_2 & (\mathrm{b}_j,\beta_j,B_j)_{1,q_1},\left(\frac{1}{2}+\sigma-\alpha;A,B\right) : (\mathrm{d}_j,\delta_j)_{1,Q_3};(f_j,F_j)_{1,Q_4} \end{array}\right) \quad (4.16)$$

By considering the same notations and conditions that corollary 2 with $\mathrm{m}_2 = n_2 = p_2 = q_2 = 0$, and $|argz_1| < \frac{1}{2}U_3''\pi, |argz_2| < \frac{1}{2}U_4''\pi$, U_3'' and U_4'' are defined by

$$U_3'' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \sum_{j=n_1+1}^{P_1}\alpha_j - \sum_{j=m_1+1}^{Q_1}\beta_j - \sum_{j=1}^{Q_2}\beta_j$$

$$\sum_{j=1}^{P_2}\alpha_j - \sum_{j=m_3+1}^{Q_3}\delta_j - \sum_{j=n_3+1}^{P_3}\gamma_j > 0 \quad (4.17)$$

$$U_4'' = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{m_2}B_j' + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \sum_{j=n_1+1}^{P_1}A_j - \sum_{j=m_1+1}^{Q_1}B_j - \sum_{j=1}^{Q_2}B_j$$

$$-\sum_{j=1}^{P_2}A_j - \sum_{j=m_4+1}^{Q_4}F_j - \sum_{j=n_4+1}^{P_4}E_j > 0 \quad (4.18)$$

Now we consider the generalized of Meijer G-function of two variables defined by Agarwal [1].

Corollary 7

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x \;\; \mathrm{G}\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right), z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$G^{m_1,n_1+1:m_3,n_3;m_4,n_4}_{P+1,Q+1:P'',Q'':P''',Q'''}\left(\begin{array}{c|c} \frac{2}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]}\, z_1 & (1+\sigma-\alpha),(a_j)_{1,p_1}\;(\mathrm{c}_j)_{1,P_3};(e_j)_{1,P_4} \\ \vdots & \vdots \\ \frac{2}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]}\, z_2 & (\mathrm{b}_j)_{1,q_1},\left(\frac{1}{2}+\sigma-\alpha\right)(\mathrm{d}_j)_{1,Q_3};(f_j)_{1,Q_4} \end{array}\right) \quad (4.19)$$

under the conditions verified by the corollary 2 and the conditions mentionned at the beginning of the corollary 3,

and $|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

www.ijrasb.com
https://doi.org/10.31033/ijrasb.9.1.3

$$\mathrm{U}_1 = \left[m_1 + n_1 + m_3 + n_3 - \tfrac{1}{2}(p_1 + q_1 + p_3 + q_3)\right] \tag{4.20}$$

and

$$\mathrm{V}_1 = \left[m_1 + n_1 + m_4 + n_4 - \tfrac{1}{2}(p_1 + q_1 + p_4 + q_4)\right] \tag{4.21}$$

Taking $\mathrm{m}_1 = 0$, we have the Aleph-function defined by Sharma [14] and Kumar [7] :

Corollary 8

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x \ \aleph\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$\begin{Bmatrix} \mathrm{c}^{-\frac{1}{2}} \\ (\mathrm{a+2b+c})^{-\frac{1}{2}} \end{Bmatrix} \aleph^{0,n_1+1:m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r:P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}$$

$$\left(\begin{matrix} \frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^A}\ \mathrm{z}_1 \\ \cdot \\ \cdot \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^B}\ \mathrm{z}_2 \end{matrix}\ \middle|\ \begin{matrix} (1+\sigma-\alpha; A, B), (a_j, \alpha_j, A_j)_{1,n_1}, [\tau_i(a_{ji}, \alpha_{ji}, A_{ji})]_{n_1+1,P_i} : \\ \cdot \\ \cdot \\ , [\tau_i(b_{ji}, \beta_{ji}, A_{ji})]_{1,Q_i}, (\tfrac{1}{2}+\sigma-\alpha; A, B) : \end{matrix}\right.$$

$$\left.\begin{matrix} (\mathrm{c}_j, \gamma_j)_{1n_3}, [\tau_{i''}(c_{ji''}, \gamma_{ji''})]_{n_3+1;P_{i''}}; (e_j, E_j)_{1n_4}, [\tau_{i'''}(e_{ji'''}, \gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{d}_j, \delta_j)_{1m_3}, [\tau_{i''}(d_{ji''}, \delta_{ji''})]_{m_3+1;Q_{i''}}; (f_j, F_j)_{1m_4}, [\tau_{i'''}(f_{ji'''}, F_{ji'''})]_{m_4+1;Q_{i'''}} \end{matrix}\right) \tag{4.22}$$

under the conditions of the corollary 4 with $\mathrm{m}_1 = 0$, $|argz_1| < \frac{1}{2}U_3''''\pi$, $|argz_2| < \frac{1}{2}U_4''''\pi$, U_3'''' and U_4'''' are defined respectively by the relations

$$U_3''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \tau_i \max_{1\le i\le 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji} + \sum_{j=1}^{Q_i}\beta_{ji}\right)$$

$$-\tau_i'' \max_{1\le i''\le 3}\left(\sum_{j=n_3+1}^{P_i''}\alpha_{ji''} + \sum_{j=m_3+1}^{Q_i''}\beta_{ji''}\right) > 0 \tag{4.23}$$

and

$$U_4''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{i=1}^{m_4}E_j + \sum_{j=1}^{n_4}F_j - \tau_i \max_{1\le i\le 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji} + \sum_{j=1}^{Q_i}\beta_{ji}\right)$$

$$-\tau_i''' \max_{1\le i'''\le 3}\left(\sum_{j=n_4+1}^{P_i'''}E_{ji'''} + \sum_{j=m_4+1}^{Q_i'''}F_{ji'''}\right) > 0 \tag{4.24}$$

Let $\tau_i, \tau_{i''}, \tau_{i'''} \to 1$, the Aleph-function cited in the corollary 5 becomes the I-function of two variables defined by Sharma and Mishra [15] and we have :

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.3

Corollary 9

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x \ \mathrm{I}\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$\left\{\begin{matrix} c^{-\frac{1}{2}} \\ (a+2b+c)^{-\frac{1}{2}}\end{matrix}\right\} I^{0,n_1+1:m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1;r:P_{i''},Q_{i''};r'':P_{i'''},Q_{i'''};r'''}$$

$$\left(\begin{matrix} \frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^A}\, z_1 \\ \cdot \\ \cdot \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^B}\, z_2 \end{matrix}\ \middle|\ \begin{matrix} (1+\sigma-\alpha; A, B), (a_j, \alpha_j, A_j)_{1,n_1}, [(a_{ji}, \alpha_{ji}, A_{ji})]_{n_1+1,P_i} : \\ \cdot \\ \cdot \\ , [(\mathrm{b}_{ji}, \beta_{ji}, A_{ji})]_{1,Q_i}, \left(\frac{1}{2}+\sigma-\alpha; A, B\right) : \end{matrix}\right.$$

$$\left.\begin{matrix} (\mathrm{c}_j, \gamma_j)_{1n_3}, [(c_{ji''}, \gamma_{ji''})]_{n_3+1;P_{i''}}; (e_j, E_j)_{1n_4}, [(e_{ji'''}, \gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{d}_j, \delta_j)_{1m_3}, [(d_{ji''}, \delta_{ji''})]_{m_3+1;Q_{i''}}; (f_j, F_j)_{1m_4}, [(f_{ji'''}, F_{ji'''})]_{m_4+1;Q_{i'''}} \end{matrix}\right) \tag{4.25}$$

Provided the notations and conditions verified by the corollary 8 with $\tau_i, \tau_{i''}, \tau_{i'''} \to 1$, U_3'''''' and U_4'''''' are defined respectively by the relations

$$U_3'''''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \max_{1\leq i\leq 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji} + \sum_{j=1}^{Q_i}\beta_{ji}\right) - \max_{1\leq i''\leq 3}\left(\sum_{j=n_3+1}^{P_i''}\alpha_{ji''} + \sum_{j=m_3+1}^{Q_i''}\beta_{ji''}\right) > 0 \quad (4.26)$$

and

$$U_4''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{i=1}^{m_4}E_j + \sum_{j=1}^{n_4}F_j - \max_{1\leq i\leq 3}\left(\sum_{j=n_1+1}^{P_i}\alpha_{ji} + \sum_{j=1}^{Q_i}\beta_{ji}\right) - \max_{1\leq i'''\leq 3}\left(\sum_{j=n_4+1}^{P_i'''}E_{ji'''} + \sum_{j=m_4+1}^{Q_i'''}F_{ji'''}\right) > 0 \quad (4.27)$$

We suppose r = r” = r”’ =0, the generalized H-function of two variables reduces to H-function of two variables defined by Gupta and Mittal [6], this gives :

Corollary 10

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}}\mathrm{d}x \ \mathrm{H}\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^A, z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)^B\right]\mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$\left\{\begin{matrix} c^{-\frac{1}{2}} \\ (a+2b+c)^{-\frac{1}{2}}\end{matrix}\right\} H^{0,n_1+1:m_3,n_3;m_4,n_4}_{P+1,Q+1:P'',Q'':P''',Q'''}\left(\begin{matrix} \frac{2^{-A}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^A}\, z_1 \\ \cdot \\ \cdot \\ \frac{2^{-B}}{[b+c+\sqrt{c}\sqrt{a+2b+v+c}]^B}\, z_2 \end{matrix}\ \middle|\ \begin{matrix} (1+\sigma-\alpha; A, B), (a_j, \alpha_j, A_j)_{1,p_1} : \\ \cdot \\ \cdot \\ (\mathrm{b}_j, \beta_j, B_j)_{1,q_1}, \left(\frac{1}{2}+\sigma-\alpha; A, B\right) : \end{matrix}\right.$$

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)
https://doi.org/10.31033/ijrasb.9.1.3

$$\left. \begin{matrix} (c_j,\gamma_j)_{1,P_3};(e_j,E_j)_{1,P_4} \\ . \\ . \\ (d_j,\delta_j)_{1,Q_3};(f_j,F_j)_{1,Q_4} \end{matrix} \right) \quad (4.28)$$

with the same notations and conditions the above corollary with r = r" = r"' =0, $|argz_1| < \frac{1}{2}U_3''\pi, |argz_2| < \frac{1}{2}U_4''\pi$, U_3'' and U_4'' are defined by

$$U_3'' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \sum_{j=n_1+1}^{P_1}\alpha_j - \sum_{j=1}^{Q_1}\beta_j - \sum_{j=1}^{Q_2}\beta_j \sum_{j=1}^{P_2}\alpha_j - \sum_{j=m_3+1}^{Q_3}\delta_j - \sum_{j=n_3+1}^{P_3}\gamma_j > 0 \quad (4.29)$$

$$U_4'' = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \sum_{j=n_1+1}^{P_1}A_j - \sum_{j=m_1+1}^{Q_1}B_j - \sum_{j=1}^{Q_2}B_j$$

$$-\sum_{j=1}^{P_2}A_j - \sum_{j=m_4+1}^{Q_4}F_j - \sum_{j=n_4+1}^{P_4}E_j > 0 \quad (4.30)$$

Let $m_1 = 0; (\alpha_j)_{1,P_1} = (A_j)_{1,P_1} = (\gamma_j)_{1,P_3} = (E_j)_{1,P_4} = (\beta_j)_{1,Q_1} = (B_j)_{1,q_1} = 1 = (\delta_j)_{1,Q_3} = (F_j)_{1,Q_4}$, the H-function of two variables is replaced by the Meijer G-function of two variables defined by Agarwal [1]. Then

Corollary 11

$$\int_0^1 \frac{x^{\alpha-1}(1-x)^{\beta}}{(ax^2+2bx+c)^{\alpha+\frac{1}{2}-\sigma}} \mathrm{d}x \ \mathrm{G}\left[z_1\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right), z_2\left(\frac{x(1-x)}{(ax^2+2bx+c)}\right)\right] \mathrm{d}x = \frac{2^{-\alpha+\sigma}\sqrt{\pi}}{\left[b+c+\sqrt{c}\sqrt{a+2b+c}\right]^{\alpha-\sigma}}$$

$$G^{0,n_1+1:m_3,n_3;m_4,n_4}_{P+1,Q+1:P'',Q'':P''',Q'''}\left(\begin{matrix} \frac{2}{[b+c+\sqrt{c}\sqrt{a+2b+v+c]}} z_1 \\ . \\ . \\ \frac{2}{[b+c+\sqrt{c}\sqrt{a+2b+v+c]}} z_2 \end{matrix} \middle| \begin{matrix} (1+\sigma-\alpha),(a_j)_{1,p_1} : (c_j)_{1,P_3};(e_j)_{1,P_4} \\ . \qquad . \\ . \qquad . \\ (b_j)_{1,q_1},(\frac{1}{2}+\sigma-\alpha) : (d_j)_{1,Q_3};(f_j)_{1,Q_4} \end{matrix} \right) \quad (4.31)$$

under the conditions verified by the corollary 2 and the conditions mentionned at the beginning of the corollary 3,

and $|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

$$U_1 = \left[n_1 + m_3 + n_3 - \tfrac{1}{2}(p_1 + q_1 + p_3 + q_3)\right] \quad (4.32)$$

and

$$V_1 = \left[n_1 + m_4 + n_4 - \tfrac{1}{2}(p_1 + q_1 + p_4 + q_4)\right] \quad (4.33)$$

Remarks

We have the same generalized finite integral with the modified generalized of I-function of two variables defined by Kumari et al. [8], see Singh and Kumar for more details [16] and the special cases, the I-function defined by Saxena [13], the I-function defined by Rathie [12], the Fox's H-function, we have the same generalized finite integrals with the incomplete aleph-function defined by Bansal et al. [3], the incomplete I-function studied by Bansal and Kumar. [2] and the incomplete Fox's H-function given by Bansal et al. [4], the Psi function defined by Pragathi et al. [9].

V. CONCLUSION

The importance of our all the results lies in their manifold generality. First by specializing the various parameters as well as variable of the generalized modified Aleph-function of two variables, we obtain a large number of results in -volving remarkably wide

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ISSN: 2349-8889
Volume-9, Issue-1 (January 2022)

www.ijrasb.com
https://doi.org/10.31033/ijrasb.9.1.3

variety of useful special functions (or product of such special functions) which are expressible in terms of the Aleph-function of two or one variables, the I-function of two variables or one variable defined by Sharma and Mishra [15] , the H-function of two or one variables , Meijer's G-function of two or one variables and hypergeometric function of two or one variables. Secondly, by specializing the parameters of this unified multipleinte - grals, we can get a large number of multiple, double or single integrals involving the modified generalized aleph-functions of two variables and the others functions seen in this document.

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