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Advances in General Topology and Its Application

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ABSTRACT

The mathematical study of continuity, connectedness and related phenomena in a broad context might be called generic topology. The real line, Euclidean spaces (including infinite dimensions), and function spaces are some of the places where these notions initially emerge. First, we must discover an abstract environment in which we can articulate findings of continuity and related notions (convergence, compactness, connectedness, and so forth) that occur in these more concrete situations. Frechet and Hausdor laid the foundations for general topology in the early 1900s. General topology is frequently referred to as point-set topology since it is based on the idea of sets. On the other side of the spectrum is algebraic topology, which applies abstract algebraic concepts to the study of de ne algebraic invariants of spaces; and, on the other hand, differential topology examines topological spaces with extra structure in order to study differentiability (basic general topology only generalizes the notion of continuous functions, not the notion of differentiable function). Since Bendse and Kikuchi's groundbreaking publication on topology optimization in 1988, topology optimization has evolved tremendously. "density," "level set," "topological derivative," 'phase field', and a slew of other terms are now being used to describe the notion. An overview, comparison, and critical analysis of the various techniques, their strengths, shortcomings, similarities and dissimilarities are presented in this work.

Keywords- Topology optimization, Density methods, Level set methods, Topological derivatives

I. INTRODUCTION

Topology is the topic of study when it comes to figuring out how objects fit together. There will soon be a presentation of concepts like closed and open forms, consistency, and homeomorphism. Geometric algebraic and arithmetic topologies were initially derived from problems in analysis and differential geometry, but they now appear in almost every branch of mathematics, including algebra, combinatorics, and logic. In general topology, a wide range of analytical and geometric issues may be studied in depth. The topology of a given object may be constructed in a basic manner. This means that patterns compatible with the original topology should be studied. Think of the smooth manifold structure of an individual's family as an example. Category homomorphisms that were also smooth mappings of the fundamental manifold are the proper morphisms to investigate. Individuals' ability to understand morphisms and come up with proofs is influenced by the relationship between these two factors. An algebraic variation of Zariski's topology is yet another unusual case. As a result of exploiting the topology of the Zariski group, it is possible to create continuous reasoning for polynomial values in practically any domain.

II. DEFINITION AND TERMS OF TOPOLOGY

Before we can get to the main topic of this review article, topology optimization, we must first define and understand the term "topology." As an etymology of the Greek noun topos, the term indicates that it relates to a particular location or area. According to Euler's Polyhedron Rule, even when arbitrary deformations are applied to three-dimensional objects like cubes, cubes, and octohedron tetrahedrons, the rule holds in three-dimensional space. ... All R3 subsets are included in topological domains (including straight lines, collections of points, and so on). A mathematical transformation or reversibly unique mapping may be used to describe any sort of distortion. Topological mapping is a term used to describe transformations from one topological domain to another that do not affect or establish new neighbourhood connections. Two domains are said to be topologically comparable if a topological mapping exists between them. It is for this reason that a domain's invariant topological attributes are those that remain constant no matter how the domain's topology changes.

III. HOW TOPOLOGY DETECTS CERTAIN PHASES OF MATTER

Geometry's topology division deals with large-scale characteristics of forms. If a coffee cup were constructed of rubber, it would be impossible to discern between a doughnut and a coffee cup, according to a popular myth. Local quantities (distances, curvature) may be measured using a geometer to identify the coffee cup from a doughnut. A topologist with apparently faulty eyes can only see that each pretzel has one hole, but at least they can tell them apart from a pretzel with two holes. [3].

3.1. A simple example from geometry

Let's start with a simple example of quantum physics. An endless line L is given a certain number of numbered balls to be inserted at n = 1, 2, etc. The "balls" shown in the illustrations below are, in fact, actual points of different sizes. We'd want to look at every possible configuration of these balls. One ball on the line L gives us everything that we need to know if we have only one. What happens if there are n = 2 balls? Two points on the line L depict a two-ball arrangement in this example. What does it look like when all of the point pairings are combined? A two-dimensional plane, of course. If we follow Descartes and name each point of L with a real number x, this becomes more evident. For any two points on L, an ordered pair (x,y) of real numbers may be equated to one. You can find all the pairs that form the x and y plane here. You may get additional information about the (x,y,z) arrangement of three points on L by starting there. A great illustration of a moduli space is the set of all possible arrangements of n balls on L. [4].

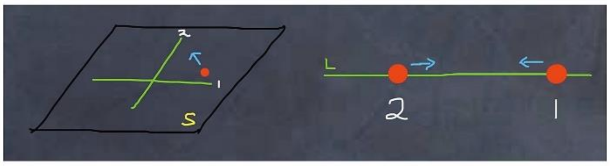

Figure 1. Motion in a moduli space

There is a shape to every possible arrangement. A one-dimensional line has n = 1, a two-dimensional plane has n = 2, and any number more than 3 has n = 3. How many deformation classes are there? For example, we may question whether every pair of configurations can be warped one to the other. To link two dissimilar configurations, as we've seen, the route is the only deformation that can do so in S. We use the notation '0(S)' to refer to S's path components. The only way to link two places in the same route component is through a path. To put it another way, a (straight line) route may link any two locations in S. A two-dimensional illustration of this may be seen in Figure 1. If all conceivable configurations are simultaneously moved to one fixed configuration, such as one where the red balls are placed on each side of L, then constrictions may be established in the plane.

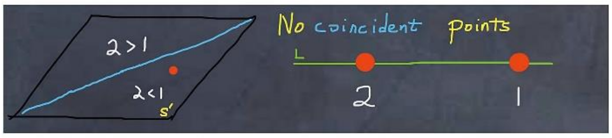

Figure 2. Moduli space of gapped configurations

We don't have any intriguing topologies as of yet. Let's add a "gap condition" to the equation. Please do not allow the balls (which are really points) to coincide. Starting with all possible configurations, we eliminate the portion of the moduli space S that corresponds to configurations in which at least two balls coincide to arrive at the gapped moduli space S'. Figure 2 shows a diagonal blue line that represents two coinciding locations of L for n = 2 balls. This line is the excluded set for n = 2. There are two paths in the complimentary space S'. There is a basic invariant that distinguishes the two halves of the journey: the sequence of integers on the line L read from left to right. As seen in Figure 2, the sequence is (2 1) and the other route component is (2 1). In other words, the number 1. As a consequence, a permutation invariant is associated with the gapped configuration. This permutation completely indicates the deformation class of the configuration [5].

3.2. Moduli spaces in quantum mechanics

Quantum physics rather than geometry is the source of our story's moduli space Since the properties of quantum systems, such as pressure, temperature, and magnetic fields, are continually changing, we might imagine a moduli space Q with points representing these quantum mechanical systems. In a quantum system, there are three components: a Hilbert space of quantum states, a Hamiltonian operator for measuring energy levels, and the G group of symmetry. There must be a gap in the energy spectrum slightly above the lowest energy for the Hamiltonian to have a range of potential energies. The letter Q may be used to represent the data in a single quantum system, as seen in Figure 3. Quantum systems with gaps may be found in the Q' subspace. There is no gap in Figure 3, which depicts a quasiquantum system. Let's get started, since there's a lot on the line.

Compute the set $\pi_0(Q')$ of deformation classes of gapped quantum systems.

Defining the system's parameters is necessary to arrive at a relevant issue. space's dimensions and symmetry group G.

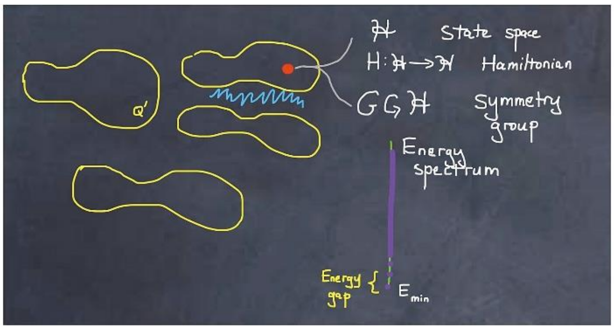

Figure 3. Moduli space of gapped quantum systems

3.3. Transition to topological field theory

According to physics, a quantum system's low-energy behaviour encodes its deformation class. The topology is not intended to be altered by high-energy fluctuations. This is a good fit for geometry's intuition. One may do a generalised Fourier analysis and decompose functions (including differential forms) based on their generalised frequency in an open space

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with a smooth curvature, referred to as a manifold. Large-scale topology can only be seen by low-frequency functions, which are locally constant on the manifold; high-frequency functions are better at detecting small-scale geometric characteristics. It is possible to create a topological invariant from the low-energy section of a quantum theory in quantum mechanics [7].

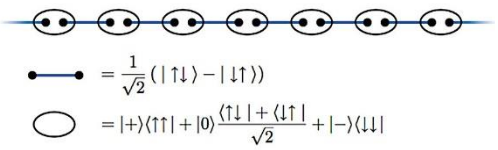

Figure 5. A simple lattice system

Taking the following step will be a great leap. Using a quantum field theory, we presume that the low-energy section of the system can be accurately described. If the initial quantum system is a quantum field theory, then this isn't such a hard jump to make. There are no lattices of degrees of freedom in these systems, and the Hamiltonian is a sum of discrete numbers. Lattice structures have a distinct character, as seen in Figure 5. In spite of this, they often have continuum bounds in which the degrees of freedom are fields: functions and other recognisable objects from smooth geometry We presume that a scale-independent quantum field theory exists at low energies, but we haven't proven it.

3.4. A mathematical framework for quantum field theory

We're getting closer to deriving a mathematical issue from the categorization problem in physics. However, in order to categorise these objects—topological quantum field theories—we must first define them mathematically. There have been mathematical representations of quantum field theory from the inception of quantum theory. A geometric axiom system for scale-invariant quantum field theories developed in the late 1980s when mathematicians started to concentrate on topological theories. The Oxford school has made significant contributions to the field. [9].

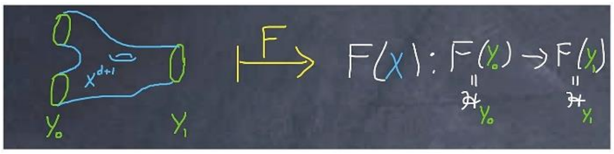

Figure 6. QFT from a bordism point of view

The basic structure is shown in Figure 6, and that is all that is required to be mentioned. Because d is the dimension of space, spacetime has a dimension of d+1. As shown in the diagram, an evolution starts at Y0 and concludes at Y1. H0 and H1 vector spaces, as well as a linear map F(X) linking them, are outputs of the field theory F throughout its development. An example of a geometric representation of quantum information is shown here. Map F is invariant to input data deformations in a topological theory of topology.

IV. STUDYING THE SHAPE OF DATA USING TOPOLOGY

When it comes to data collection and storage, it's known as the "data explosion," and it's happening all over the world in fields such as research, engineering, business, and the government. Large data sets are all around us, and they have important implications for our daily lives and for society as a whole. This is something we are reminded of every time we open the news or a computer. During the last fifteen years, the field of topological data analysis (TDA) has seen an explosion of interest and activity, resulting in both useful new methods for analysing data as well as delightful mathematical surprises. TDA has been used in cancer, astronomy, neurology, image processing, and biophysics, among other fields of study. TDA's primary purpose is to use topology, one of the most important fields of mathematics, to build tools for analysing geometric aspects in data. Data" is merely a limited collection of points in space that we refer to as "points." [10].

Figure 7: A data set with three clusters;

Consider the data shown in Figure 7 as a first example. Three separate clusters may be seen in the data. The first form of geometric feature we look at in TDA are these kinds of clusters. Our goal is to identify and count the number of different clusters in the data. When the data is noisy, as seen in Figure 8, we would want to be able to accomplish this.

Topological data analysis also examines "loops" as a geometric aspect of data. Loops are shown in Figure 8 of the dataset. If a data collection is damaged by noise, as shown in Figure 10, we still want to be able to recognise loops.

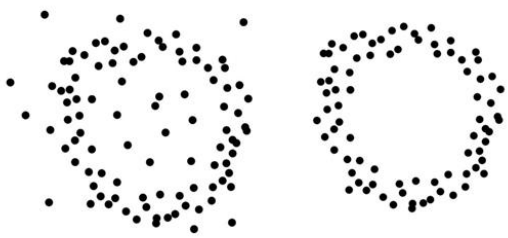

Figure 8: A data set with a loop; Figure 10: A data set with a noisy loop

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The third kind of geometric feature we look at in TDA is a "tendril." A centre nucleus is linked to three outer tendrils, as seen in Figure 11. A dataset constructed in this way requires identifying and counting the tendrils, as well as segmenting the data into its numerous tendrils.

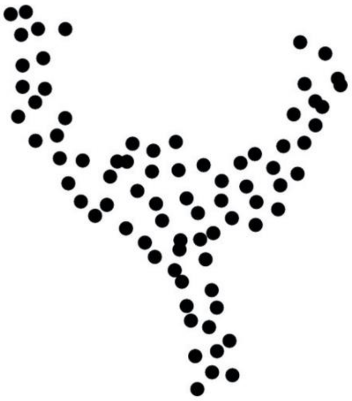

Figure 9: A data set with three tendrils emanating from a central core

In TDA research, the goal is to create tools for detecting and visualizing these sorts of geometric characteristics, as well as methods for assessing the statistical importance of these features in randomly collected data. The emphasis is on building tools for analyzing geometric aspects in high-dimensional data since so much of the data generated in scientific applications is in high-dimensional environments.

When it comes to data analysis, we are interested in using topology's capabilities for calculating the number of holes and components that a geometric object has. There are no holes in the data set, hence the number of holes and components of X will not tell us anything important about the geometric characteristics in the data set X of n points in space.

As a result, instead of explicitly examining the topological features of X, we will focus on the qualities of a "thickening" of it.

I'm going to go into great depth about this. To begin, let's pretend there is just one possible collection of X points in the plane (2-D space). Let δ be a positive number, and let $T(X, \delta)$ be the set of all points in the plane within distance δ from some point in X; we think of $T(X, \delta)$ as a "thickening" of the data set X.

"Figure 9 shows $T(X_1, \delta_1)$ The original data X1 is shown in black and red for some positive integer 1. Let X2 be the data set shown in Figure 9. What's seen in Figure 12 $T(X_2, \delta_2)$ in red, for some choice of positive number δ_2, together with X_2 in black. For especially nice data sets X and good choices of δ, the clusters in X will correspond to components of $T(X, \delta)$ and the loops in X will correspond to holes in $T(X, \delta)$. For instance, in Figure 9 the clusters in X_1 correspond to 'the components of $T(X_1, \delta_1)$, and in Figure 10) the loop in X_2 corresponds to the hole in $T(X_2, \delta_2)$".

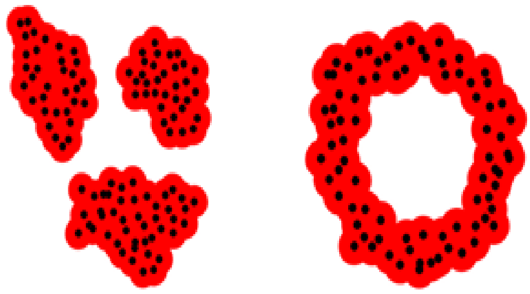

Figure 10:T(X1, δ1), for some choice of δ1, is shown in red; X1 is shown in black.; Figure 13: T(X2, δ2), for some choice of δ2, is shown in red; X2, is shown in red; X2 is shown in black

As a result, more complex variations of this fundamental method are necessary to cope with the majority of data seen in reality. Recent research on TDA has concentrated on producing new variants. Consideration of the topological qualities of the complete family of objects is a key notion in this approach. $T(X, \delta)$ as δ varies than it is to consider the topological properties of $T(X, \delta)$ for a single choice of δ. This is the idea behind persistent homology, a key technical tool in TDA.

V. IDENTIFYING ORDER IN COMPLEX SYSTEMS

Nature may have been organized in a way that cannot be understood or that can only be understood from a different perspective. In order to comprehend these notions, mathematicians often turn to the most exact tool we have: a mathematical analysis.

Calculus and topology, two qualitative branches of mathematics, have received much less attention in mathematical applications than their quantitative counterparts (the study of properties that are preserved in geometric figures despite continuous deformations). Robert MacPherson, Hermann Weyl Professor at the School of Mathematics, is eager to see whether topology may be used outside of high-energy physics (such as Chern-Simons theory). [11]

One of Princeton's initial professors in 1932 and the driving force behind Princeton's position as the world leader in topology was Oswald Veblen. MacPherson launched graduate engineering seminars at Princeton University some years ago with the assumption that topological thinking may be applied to tackle problems relating to materials. Professors MacPherson and Srolovitz of Princeton University's Department of Mechanical and Aerospace Engineering found John von Neumann's two-dimensional solution to a problem that had been open since 1952 two years ago. [12].

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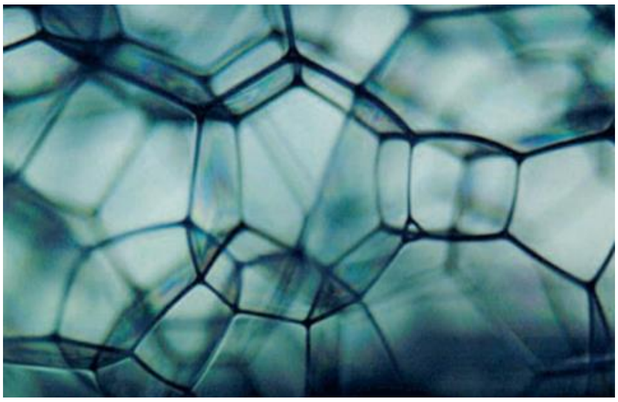

The work of MacPherson and a partner on models of three-dimensional grain formation has ramifications for several materials, including foam.

Foams and metals both have three-dimensional cellular structures that affect critical material qualities such as strength and magnetism, whereas soft materials like metals and ceramics have more sophisticated fluid behaviour, such as breaking waves. Since the three-dimensional von Neumann formula is relevant for this, An investigation of these concepts continues at the Institute, where they are examined. Jeremy Mason, a postdoctoral expert in hard materials from the Massachusetts Institute of Technology, and Randall Kamien, a professor of soft materials from the University of Pennsylvania, will join the School of Mathematics in the forthcoming academic year. [13].

REFERENCES

[1] Frédéric Chazal, Leonidas J. Guibas, Steve Y. Oudot, and Primoz Skraba, "Persistence-Based Clustering in Riemannian Manifolds," in Proceedings of the Twenty-Seventh Annual ACM Symposium on Computational Geometry (Association for Computing Machinery, 2011), 97–106.

[2] Anil K. Jain, M. Narasimha Murty, and Patrick J. Flynn, "Data Clustering: A Review," ACM Computing Surveys (CSUR) 31, no. 3 (1999): 264–323.

[3] Marcio Gameiro, Yasuaki Hiraoka, Shunsuke Izumi, MiroslavKramar, Konstantin Mischaikow, and Vidit Nanda, "Topological Measurement of Protein Compressibility via Persistence Diagrams," MI Preprint Series 2012-6, Faculty of Mathematics, Kyushu University, 2012.

[4] Gunnar Carlsson, Tigran Ishkhanov, Vin De Silva, and Afra Zomorodian, "On the Local Behavior of Spaces of Natural Images," International Journal of Computer Vision 76, no. 1 (2008): 1–12.

[5] Monica Nicolau, Arnold J. Levine, and Gunnar Carlsson, "Topology-Based Data Analysis Identifies a Subgroup of Breast Cancers with a Unique Mutational Profile and Excellent Survival," Proceedings of the National Academy of Sciences 108, no. 17 (2011): 7265–70.

[6] Gurjeet Singh, Facundo Mémoli, and Gunnar Carlsson, "Topological Methods for the Analysis of High-Dimensional Data Sets and 3-D Object Recognition," in Eurographics Association Symposium on Point-Based Graphics 22 (The Eurographics Association, 2007).

[7] Gunnar Carlsson, "Topology and Data," Bulletin of the American Mathematical Society 46, no. 2 (2009): 255–308.

[8] Herbert Edelsbrunner and John L. Harer, "Persistent Homology: A Survey," in Surveys on Discrete and Computational Geometry: Twenty Years Later: AMS-IMS-SIAM Joint Summer Research Conference, June 18–22, 2006, Snowbird, Utah 453 (American Mathematical Society, 2008), 257.

[9] Robert Ghrist, "Barcodes: The Persistent Topology of Data," Bulletin of the American Mathematical Society 45, no. 1 (2008): 61.

[10] Herbert Edelsbrunner and John L. Harer, Computational Topology: An Introduction (American Mathematical Society, 2010).

[11] The work of Michael Lesnick, Member (2012–13) in the School of Mathematics, focuses on the theoretical foundations of topological data analysis.

[12] Daniel S. Freed, IBM Einstein Fellow in the Schools of Mathematics and Natural Sciences (2015), works on aspects of topological field theory. His current projects are broadly related to six-dimensional superconformal field theory as well as phases in condensed matter physics. Freed is Professor in the Department of Mathematics at the University of Texas at Austin.

[13] Kelly Devine Thomas is Editorial Director at the Institute for Advanced Study. The Institute Letter Summer 2009

