International Journal for Research in Applied Sciences and Biotechnology
ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)
www.ijrasb.com
https://doi.org/10.31033/ijrasb.8.2.1

Finite Integral Involving the Modified Generalized Aleph-Functionof Two Variables and Elliptic Integral of First Species I

Frédéric Ayant[1] and Prvindra Kumar[2]
[1]Teacher in High School, FRANCE
[2]Department of Mathematics, D.J College, Baraut, INDIA

[1]Corresponding Author: prvindradjc@gmail.com

ABSTRACT

In the present paper, we evaluate the general finite integral involving the elliptic integrals of first species and the generalized modified Aleph- function of two variables. At the end, we shall see several corollaries and remarks.

Keywords- Generalized modified Aleph-function of two variables, generalized modified I-function of two variables, generalized modified H-function of two variables, generalized modified Meijer-function of two variables, Aleph-function of two variables, I-function of two variables, H- function of two variables, Meijer G-function of two variables, Two Mellin-Barnes integrals contour, elliptic integrals of first species.

2010 Mathematics Subject Classification: 33C05, 33C60.

I. INTRODUCTION

Recently Aleph-function of two variables has been introduced and studied by Sharma [14], Kumar [8], it's an extension of I-function of two variables defined Sharma and Mishra [10] which is a generalization of the H-function of two variables due to Gupta and Mittal. [7]. On the other hand Prasad and Prasad [11] have defined the modified generalized H-function of two variables. In this paper, first, we introduce and study the generalized modified Aleph-function of two variables. This function unify the Aleph-function of two variables and the modified of generalized H-function of two variables. Later, we calculate a finite generalized integral involving this function. At the end, we will given several special cases and remarks.

The double Mellin-Barnes integrals contour occurring in this paper will be referred to as the generalized function of two variables throughout our present study and will be defined and represented as follows:

$$\aleph_1(z_1,z_2) = \aleph^{m_1,n_1;m_2,n_2;m_3,n_3;m_4,n_4}_{P_i,Q_i,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|l} \mathrm{z}_1 & (\mathrm{a}_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot & \\ \cdot & \\ \mathrm{z}_2 & (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i}: \end{array}\right.$$

$$\left.\begin{array}{l} (\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}:(\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)x^s y^t \mathrm{d}s\mathrm{d}t, \tag{1.1}$$

where,

$$\phi(s,t) = \frac{\prod_{j=1}^{n_1}\Gamma(1-a_j+\alpha_j s+A_j t)\prod_{j=1}^{m_1}\Gamma(b_j-\beta_j s-B_j t)}{\sum_{i=1}^{r}\tau_i\left[\prod_{j=m_1+1}^{Q_i}\Gamma(1-b_{ji}+\beta_{ji}s+B_{ji}t)\prod_{n_1+1}^{P_i}\Gamma(a_{ji}-\alpha_{ji}s-A_{ji}t)\right]}$$

$$\frac{\prod_{j=1}^{n_2}\Gamma(1-a'_j+\alpha'_j s-A'_j t)\prod_{j=1}^{m_2}\Gamma(b'_j-\beta'_j s+B'_j t)}{\sum_{i'=1}^{r'}\tau_{i'}\left[\prod_{j=m_2+1}^{Q_{i'}}\Gamma(1-b'_{ji'}+\beta'_{ji'}s-B'_{ji'}t)\prod_{n_2+1}^{P_{i'}}\Gamma(a'_{ji'}-\alpha'_{ji'}s+A'_{ji'}t)\right]} \tag{1.2}$$

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)
https://doi.org/10.31033/ijrasb.8.2.1

$$\theta_1(s) = \frac{\prod_{j=1}^{m_3}\Gamma(d_j-\delta_j s)\prod_{j=1}^{n_3}\Gamma(1-c_j+\gamma_j s)}{\sum_{i''=1}^{r''}\tau_{i''}\left[\prod_{j=m_3+1}^{Q_{i''}}\Gamma(1-d_{ji''}+\delta_{ji''}s)\prod_{j=n_3+1}^{P_{i''}}\Gamma(c_{ji''}-\gamma_{ji''}s)\right]} \quad (1.3)$$

$$\theta_2(t) = \frac{\prod_{j=1}^{m_4}\Gamma(f_j-F_j t)\prod_{j=1}^{n_4}\Gamma(1-e_j+E_j t)}{\sum_{i'''=1}^{r'''}\tau_{i'''}\left[\prod_{j=m_4+1}^{Q_{i'''}}\Gamma(1-f_{ji'''}+F_{ji'''}t)\prod_{j=n_4+1}^{P_{i'''}}\Gamma(e_{ji'''}-E_{ji'''}t)\right]} \quad (1.4)$$

where Z_1 and Z_2 (real or complex) are not equal to zero and an empty product is interpreted as unity and the quantities P_i, $P_{i'}$, $P_{i''}$, $P_{i'''}$, Q_i, $Q_{i'}$, $Q_{i''}$, $Q_{i'''}$, m_1, m_2, m_3, m_4, n_1, n_2, n_3, n_4 are non-negative integers such that $Q_i>0$, $Q_{i'}>0$, $Q_{i''}>0$, $Q_{i'''}>0$, τ_i, $\tau_{i'}$ $\tau_{i''}$ $\tau_{i'''}$ >0 (i= 1……r), (i'= 1……..r') (i'' = 1…..r'') (i''' = 1……r''')

All the A's, α's, B's, β's, A''s, B''s, α''s, β''s, γ's,δ's, E's and F's are assumed to be positive quantities for standardization purpose; the definition of generalized Aleph-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the s-plane and runs from - ω∞ to + ω∞ if necessary, to ensure that the poles of $\Gamma(b_j - \beta_j s - B_j t)$(j=1,……$m_1$),$\Gamma(d_j - \delta_j$)(j=1,……$m_3$) and $\Gamma(b'_j - \beta'_j s - B'_j t)$(j=1,……$m_2$) are to the right and all the poles of $\Gamma(1 - a_j + \alpha_j s + A_j t)$ (j=1,……n_1), $\Gamma(1 + c_j s + \gamma s)$ (j=1,……n_3) and $\Gamma(1 - a'_j + \alpha'_j s - A'_j t)$ (j=1,……n_2) lie to the left of L_1. The contour L_2 is in the t-plane and runs from - ω∞ to + ω∞ with loops, if necessary, to ensure that the poles of $\Gamma(b_j - \beta_j s - B_j t)$(j=1,……$m_1$), $\Gamma(f_j - F_j t)$ (j=1,……m_4), $\Gamma(1 - a'_j + \alpha'_j s - A'_j t)$ (j=1,……n_2) are to the right and all the poles of $\Gamma(1 - a_j + \alpha_j s + A_j t)$ (j=1,……n_1), $\Gamma(1 - e_j + E_j t)$ (j=1,……n_3) and $\Gamma(b'_j - \beta'_j s + B'_j t)$(j=1,……$m_2$) lie to the left of L_2. The poles of the integrand are assumed to be simple.

The function defined by the equation (3.1) is analytic function of if z_1 and z_2 if

$$U_1 = \tau_i\sum_{j=1}^{P_i}\alpha_{ji} - \tau_i\sum_{j=1}^{Q_i}\beta_{ji} + \tau_{i'}\sum_{j=1}^{P_{i'}}\alpha_{ji'} - \tau_{i'}\sum_{j=1}^{Q_{i'}}\beta_{ji'} + \tau_{i''}\sum_{j=1}^{P_{i''}}\gamma_{ji''} - \tau_{i''}\sum_{j=1}^{Q_{i''}}\delta_{ji''} < 0 \quad (1.5)$$

$$U_2 = \tau_i\sum_{j=1}^{P_i}A_{ji} - \tau_i\sum_{j=1}^{Q_i}B_{ji} + \tau_{i'}\sum_{j=1}^{P_{i'}}A_{ji'} - \tau_{i'}\sum_{j=1}^{Q_{i'}}B'_{ji'} + \tau_{i'''}\sum_{j=1}^{P_{i'''}}E_{ji'''} - \tau_{i'''}\sum_{j=1}^{Q_{i'''}}F_{ji'''} < 0 \quad (1.6)$$

The integral defined by (2.1) is converges absolutely, if

$$U_3 = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_2}\alpha'_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{m_2}\beta'_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \tau_i\sum_{j=n_1+1}^{P_i}\alpha_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}\beta_{ji} - \tau_{i'}\sum_{j=m_2+1}^{Q_{i'}}\beta_{ji'}$$
$$-\tau_{i'}\sum_{j=n_2+1}^{P_{i'}}\alpha_{ji'} - \tau_{i''}\sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''} - \tau_{i''}\sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''} > 0 \quad (1.7)$$

$$U_4 = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{n_2}A'_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{m_2}B'_j + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \tau_i\sum_{j=n_1+1}^{P_i}A_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}B_{ji} - \tau_{i'}\sum_{j=m_2+1}^{Q_{i'}}B_{ji'}$$
$$-\tau_{i'}\sum_{j=n_2+1}^{P_{i'}}A_{ji'} - \tau_{i'''}\sum_{j=m_4+1}^{Q_{i'''}}F_{ji'''} - \tau_{i'''}\sum_{j=n_4+1}^{P_{i'''}}E_{ji'''} > 0 \quad (1.8)$$

$$|arg z_1| < \frac{\pi}{2}U_3 \text{ and } |arg z_2| < \frac{\pi}{2}U_4$$

We may establish the asymptotic behavior in the following convenient form; see B.L.J. Braaksma [5]:

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)

www.ijrasb.com https://doi.org/10.31033/ijrasb.8.2.1

$$\aleph(z_1, z_2) = 0(\ |z_1|^{\alpha_1},\ |z_r|^{\alpha_2}\ ),\ max(\ |z_1|,\ |z_2|\ ) \to 0$$

$$\aleph(z_1, z_2) = 0(\ |z_1|^{\beta_1}, |z_2|^{\beta_2}\ ),\ min(\ |z_1|, |z_2\ |\ ) \to \infty\ :$$

where $\alpha_1 = \min_{1 \leqslant j \leqslant m_3} Re\left[\left(\frac{d_j}{\delta_j}\right)\right]$ and $\alpha_2 = \min_{1 \leqslant j \leqslant m_4} Re\left[\left(\frac{f_j}{F_j}\right)\right]$

where $\beta_1 = \max_{1 \leqslant j \leqslant n_3} Re\left[\left(\frac{c_j - 1}{\gamma_j}\right)\right]$ and $\beta_2 = \max_{1 \leqslant j \leqslant n_4} Re\left[\left(\frac{e_j - 1}{E_j}\right)\right]$

The complete elliptic integrals of first species are defined by see Whittaker and Watson ([18], p. 515).

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - k^2 sin^2\theta}} = \int_0^1 \frac{dt}{\sqrt{(1 - t^2)(1 - k^2t^2)}} \tag{1.9}$$

II. REQUIRED INTEGRAL

In this section, we give a general finite integral, see Brychkov ([6], 4.21.3 Eq.11 page 270).

Lemma

$$\int_0^1 x^{u-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x})dx = \frac{\pi a \Gamma^2\left(u+\frac{1}{2}\right)}{\Gamma^2(u+1)}\ {}_4F_3\left(\begin{matrix} \frac{1}{2},\ 1, u+\frac{1}{2},\ u+\frac{1}{2} \\ \cdot\ \cdot\ \cdot \\ \frac{3}{2},\ u+1, u+1 \end{matrix} \middle|\ a^2 \right) \tag{2.1}$$

where

$Re(s) > -\frac{1}{2}, |arg(1 - a^2)| < \pi\cdot$

III. MAIN INTEGRAL

In this section, we study a generalization of the finite integral involving the modified of generalized Aleph-funtion of two variables. We have the unified finite integral.

Theorem

$$\int_0^1 x^{u-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x})\,\aleph(z_1x^A, z_2x^B)dx = \pi a \sum_{n'=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'} n'!}\, a^{2n'}$$

$$\aleph^{m_1,n_1+2;m_2,n_2:m_3,n_3;m_4,n_4}_{P_i+2,Q_i+2,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_2 \end{matrix}\ \middle|\ \begin{matrix} \mathbf{A}_1, (a_j, \alpha_j, A_j)_{1,n_1}, [\tau_i(a_{ji}, \alpha_{ji}, A_{ji})]_{n_1+1,P_i} : \\ \cdot \\ \cdot \\ (b_j, \beta_j, B_j)_{1,m_1}, [\tau_i(b_{ji}, \beta_{ji}, B_{ji})]_{m_1+1,Q_i}, \mathbf{B}_1 : \end{matrix}\right.$$

$$\left.\begin{matrix} (a'_j, \alpha'_j, A_j)_{1,n_2}, [\tau_{i'}(a_{ji'}, \alpha_{ji'}, A_{ji'})]_{n_2+1,P_{i'}} : (c_j, \gamma_j)_{1n_3}, [\tau_{i''}(c_{ji''}, \gamma_{ji''})]_{n_3+1;P_{i''}}; (e_j, E_j)_{1n_4}, [\tau_{i'''}(e_{ji'''}, \gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (b'_j, \beta'_j, B_j)_{1,m_2}, [\tau_{i'}(b_{ji'}, \beta_{ji'}, B_{ji'})]_{m_2+1,Q_{i'}} : (d_j, \delta_j)_{1m_3}, [\tau_{i''}(d_{ji''}, \delta_{ji''})]_{m_3+1;Q_{i''}}; (f_j, F_j)_{1m_4}, [\tau_{i'''}(f_{ji'''}, F_{ji'''})]_{m_4+1;Q_{i'''}} \end{matrix}\right) \tag{3.1}$$

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)

https://doi.org/10.31033/ijrasb.8.2.1

where,

$$\mathbf{A}_1 = \left(\tfrac{1}{2} - u - n'; A, B\right), \left(\tfrac{1}{2} - u - n'; A, B\right); \mathbf{B}_1 = (-u - n'; A, B), (-u - n'; A, B) \tag{3.2}$$

Provided

$$Re(u) > -\tfrac{1}{2}, |arg(1-a^2)| < \pi. -\frac{1}{2} < Re(u) + (A+B) \min_{1\leqslant j\leqslant m_3} Re\left(\frac{d_j}{\delta_j}\right), -\frac{1}{2} < Re(u) + (A+B) \min_{1\leqslant j\leqslant m_4} Re\left(\frac{f_j}{F_j}\right)$$

$A, B > 0, |argz_1| < \frac{1}{2}U_3\pi, |argz_2| < \frac{1}{2}U_4\pi$, U_3 and U_4 are defined respectively by the equations (1.7) and (1.8).

Proof

To prove the theorem, expressing the modified generalized Aleph-function of two variables in double Mellin-Barnes contour integrals with the help of (1.1) and interchanges the order of integrations, which is justifiable due to absolute convergence of the integral involved in the process. Now collecting the power of x, we note the left hand side of the equation (3.1) J, we obtain:

$$J = \int_0^1 x^{u-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x}) \aleph(z_1 x^A, z_2 x^B) dx =$$

$$\frac{1}{(2\pi\omega)^2} \int_{L_1}\int_{L_2} \phi(s,t)\theta_1(s)\theta_2(t) z_1^s z_2^t \int_0^1 x^{u+As+Bt-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x})\, dx\, ds dt \tag{3.3}$$

We use the lemma, this gives:

$$J = \int_0^1 x^{u-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x}) \aleph(z_1 x^A, z_2 x^B) dx =$$

$$\frac{1}{(2\pi\omega)^2} \int_{L_1}\int_{L_2} \phi(s,t)\theta_1(s)\theta_2(t) z_1^s z_2^t \frac{\pi a \Gamma^2\left(u+As+Bt+\frac{1}{2}\right)}{\Gamma^2(u+As+Bt+1)}$$

$${}_4F_3\left(\begin{array}{c} \frac{1}{2}, 1, u+As+Bt+\frac{1}{2}, u+As+Bt+\frac{1}{2} \\ \cdots \\ \frac{3}{2}, u+As+Bt+1, u+As+Bt+1 \end{array} \middle| a^2 \right) ds\, dt \tag{3.4}$$

We replace the Gauss hyper geometric function by the $\sum_{n=0}^{\infty}$ series, (see Slater [17]), under the hypothesis, we can interchanged this series and the (s, t) - integrals, we have:

$$J = \int_0^1 x^{u-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x}) \aleph(z_1 x^A, z_2 x^B) dx = \pi a \sum_{n'=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'} n'!}$$

$$\frac{1}{(2\pi\omega)^2} \int_{L_1}\int_{L_2} \phi(s,t)\theta_1(s)\theta_2(t) z_1^s z_2^t \frac{\Gamma^2\left(u+As+Bt+\frac{1}{2}\right)}{\Gamma^2(u+As+Bt+1)} \frac{\left(u+As+Bt+\frac{1}{2}\right)_{n'}^2}{(u+As+Bt+1)_{n'}^2} a^{2n'}\, ds\, dt \tag{3.5}$$

Now we apply $\Gamma(\alpha)(\alpha)_n = \Gamma(\alpha+n)$ the relation where a≠ 0, -1, -2…..This gives:

$$J = \int_0^1 x^{u-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x}) \aleph(z_1 x^A, z_2 x^B) dx = \pi a \sum_{n'=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'} n'!} a^{2n'}$$

$$\frac{1}{(2\pi\omega)^2} \int_{L_1}\int_{L_2} \phi(s,t)\theta_1(s)\theta_2(t) z_1^s z_2^t \frac{\Gamma^2\left(u+As+Bt+\frac{1}{2}+n'\right)}{\Gamma^2(u+As+Bt+1+n')}\, ds\, dt \tag{3.6}$$

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)

https://doi.org/10.31033/ijrasb.8.2.1

We interpret these double contour integrals of the modified generalized Aleph-function of two variables, we have the desired result. In the following section, we cite several particular cases and remarks.

IV. SPECIAL CASES

Let r_i, r_i', r_i'', r_i''' →1 the generalized modified Aleph-function of two variables reduces to modified of generalized I-function of two variables, this gives

Corollary 1.

$$\int_0^1 x^{u-1}\ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right)K(\sqrt{1-x})\,I(z_1x^A,z_2x^B)\mathrm{d}x=\pi a\sum_{n'=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'}n'!}\,a^{2n'}$$

$$I^{m_1,n_1+2;m_2,n_2:m_3,n_3;m_4,n_4}_{P_i+2,Q_i+2;r;P_{i'},Q_{i'};r';P_{i''},Q_{i''};r'':P_{i'''},Q_{i'''};r'''}\left(\begin{array}{c|l} z_1 & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot & \qquad\cdot \\ \cdot & \qquad\cdot \\ z_2 & (b_j,\beta_j,B_j)_{1,m_1},[(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1: \end{array}\right.$$

$$\left.\begin{array}{l} (a'_j,\alpha'_j,A_j)_{1,n_2},[(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (c_j,\gamma_j)_{1n_3},[(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (b'_j,\beta'_j,B_j)_{1,m_2},[(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}\ (d_j,\delta_j)_{1m_3},[(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right) \quad (4.1)$$

By respecting the conditions and notations verified by the fundamental theorem where $\tau_i,\tau_{i'},\tau_{i''},\tau_{i'''}\to 1$ and $A,B>0$, $|argz_1|<\frac{1}{2}U_3'\pi$, $|argz_2|<\frac{1}{2}U_4'\pi$, U_3' and U_4' are defined by

$$U_3'=\sum_{j=1}^{n_1}\alpha_j+\sum_{j=1}^{n_2}\alpha'_j+\sum_{j=1}^{m_1}\beta_j+\sum_{j=1}^{m_2}\beta'_j+\sum_{j=1}^{n_3}\gamma_j+\sum_{j=1}^{m_3}\delta_j-\sum_{j=n_1+1}^{P_i}\alpha_{ji}-\sum_{j=m_1+1}^{Q_i}\beta_{ji}-\sum_{j=m_2+1}^{Q_{i'}}\beta_{ji'}$$
$$-\sum_{j=n_2+1}^{P_{i'}}\alpha_{ji'}-\sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''}-\sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''}>0 \quad (4.2)$$

$$U_4'=\sum_{j=1}^{n_1}A_j+\sum_{j=1}^{n_2}A'_j+\sum_{j=1}^{m_1}B_j+\sum_{j=1}^{m_2}B'_j+\sum_{j=1}^{n_4}E_j+\sum_{j=1}^{m_4}F_j-\sum_{j=n_1+1}^{P_i}A_{ji}-\sum_{j=m_1+1}^{Q_i}B_{ji}-\sum_{j=m_2+1}^{Q_{i'}}B_{ji'}$$
$$-\sum_{j=n_2+1}^{P_{i'}}A_{ji'}-\sum_{j=m_4+1}^{Q_{i'''}}F_{ji'''}-\sum_{j=n_4+1}^{P_{i'''}}E_{ji'''}>0 \quad (4.3)$$

We consider the above corollary and we suppose r= r'=r''=r'''=1 we obtain the generalized modified H-function defined by Prasad and Prasad [11] and the following integral.

Corollary 2.

$$\int_0^1 x^{u-1}\ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right)K(\sqrt{1-x})\,H(z_1x^A,z_2x^B)\mathrm{d}x=\pi a\sum_{n'=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'}n'!}\,a^{2n'}$$

$$H^{m_1,n_1+2;m_2,n_2:m_3,n_3;m_4,n_4}_{P+2,Q+2:P',Q':P'',Q'':P''',Q'''}\left(\begin{array}{c|l} z_1 & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,P_1}:\ (a'_j,\alpha'_j,A_j)_{1,P_2}:(c_j,\gamma_j)_{1,P_3};(e_j,E_j)_{1,P_4} \\ \cdot & \qquad\cdot \\ \cdot & \qquad\cdot \\ z_2 & (b_j,\beta_j,B_j)_{1,Q_1},\mathbf{B}_1:\ (b'_j,\beta'_j,B_j)_{1,Q_2}:(d_j,\delta_j)_{1,Q_3};(f_j,F_j)_{1,Q_4} \end{array}\right) \quad (4.4)$$

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)

https://doi.org/10.31033/ijrasb.8.2.1

under the condtions and notations satisfied by the above corollary and r = r' = r" = r"' =1, $A, B > 0$,and $|argz_1| < \frac{1}{2}U_3''\pi, |argz_2| < \frac{1}{2}U_4''\pi$, U_3'' and U_4'' are defined by

$$U_3'' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_2}\alpha_j' + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{m_2}\beta_j' + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \sum_{j=n_1+1}^{P_1}\alpha_j - \sum_{j=m_1+1}^{Q_1}\beta_j - \sum_{j=m_2+1}^{Q_2}\beta_j$$

$$\sum_{j=n_2+1}^{P_2}\alpha_j - \sum_{j=m_3+1}^{Q_3}\delta_j - \sum_{j=n_3+1}^{P_3}\gamma_j > 0 \tag{4.5}$$

$$U_4'' = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{n_2}A_j' + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{m_2}B_j' + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \sum_{j=n_1+1}^{P_1}A_j - \sum_{j=m_1+1}^{Q_1}B_j - \sum_{j=m_2+1}^{Q_2}B_j$$

$$- \sum_{j=n_2+1}^{P_2}A_j - \sum_{j=m_4+1}^{Q_4}F_j - \sum_{j=n_4+1}^{P_4}E_j > 0 \tag{4.6}$$

Taking $(\alpha_j)_{1,P_1} = (A_j)_{1,P_1} = (\alpha_j')_{1,P_2} = (A_j')_{1,P_2} = (\gamma_j)_{1,P_3} = (E_j)_{1,P_4} = (\beta_j)_{1,Q_1} = (B_j)_{1,q_1} = (\beta_j')_{1,Q_2} = 1 =$ $=A=B = (B_j')_{1,Q_2} = (\delta_j)_{1,Q_3} = (F_j)_{1,Q_4}$, the modified generalized H-function of two variables is replaced by the generalized modified of Meijer G-function of two variables defined by Agarwal [1], we have

Corollary 3.

$$\int_0^1 x^{u-1}\ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right)K(\sqrt{1-x})G(z_1x, z_2x)\mathrm{d}x = \pi a\sum_{n'=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'}n'!}a^{2n'}$$

$$G^{m_1,n_1+2:m_2,n_2:m_3,n_3;m_4,n_4}_{P+2,Q+2:P',Q':P'',Q'':P''',Q'''}\left(\begin{array}{c|c} z_1 & A_1', (a_j)_{1,P_1} : (a_j')_{1,P_2} : (c_j)_{1,P_3}; (e_j)_{1,P_4} \\ \cdot & \cdot \quad \cdot \quad \cdot \\ \cdot & \cdot \quad \cdot \quad \cdot \\ z_2 & (b_j)_{1,Q_1}, B_1' : (b_j')_{1,Q_2} : (d_j)_{1,Q_3}; (f_j)_{1,Q_4} \end{array}\right) \tag{4.7}$$

under the conditions verified by the corollary 2 and the conditions mentionned at the beginning of the corollary 3,

$$A_1' = \left(\tfrac{1}{2} - u - n'\right), \left(\tfrac{1}{2} - u - n'\right);\ B_1' = (-u - n'), (-u - n') \tag{4.8}$$

and $|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

$$U_1 = \left[m_1 + n_1 + m_2 + n_2 + m_3 + n_3 - \tfrac{1}{2}(p_1 + q_1 + p_2 + q_2 + p_3 + q_3)\right] \tag{4.9}$$

and

$$V_1 = \left[m_1 + n_1 + m_2 + n_2 + m_4 + n_4 - \tfrac{1}{2}(p_1 + q_1 + p_2 + q_2 + p_4 + q_4)\right] \tag{4.10}$$

Let $m_2 = n_2 = P_{i'} = Q_{i'} = 0$ the modified of generalized Aleph-function of two variables reduces to generalized Aleph-function of two variables, this gives:

Corollary 4.

$$\int_0^1 x^{u-1}\ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right)K(\sqrt{1-x})\aleph(z_1x^A, z_2x^B)\mathrm{d}x = \pi a\sum_{n'=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'}n'!}a^{2n'}$$

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)
https://doi.org/10.31033/ijrasb.8.2.1

$$\aleph^{m_1,n_1+2:m_3,n_3;m_4,n_4}_{P_i+2,Q_i+2,\tau_i;r:P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|l} z_1 & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot & \quad\cdot \\ \cdot & \quad\cdot \\ z_2 & (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1: \end{array}\right.$$

$$\left.\begin{array}{l}(\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}}\end{array}\right) \tag{4.11}$$

Provided the conditions verified by the fundamental theorem with the conditions $m_2 = n_2 = P_{i'} = Q_{i'} = 0$ and $A, B > 0, |argz_1| < \frac{1}{2}U_3'''\pi, |argz_2| < \frac{1}{2}U_4'''\pi$, U_3''' and U_4''' are defined respectively by the relations

$$U_3''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \tau_i\sum_{j=n_1+1}^{P_i}\alpha_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}\beta_{ji} - \tau_{i''}\sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''} - \tau_{i''}\sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''} > 0 \tag{4.12}$$

and

$$U_4''' = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \tau_i\sum_{j=n_1+1}^{P_i}A_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}B_{ji} - \tau_{i'}\sum_{j=m_2+1}^{Q_{i'}}B_{ji'}$$

$$-\tau_{i''}\sum_{j=m_4+1}^{Q_{i'''}}F_{ji'''} - \tau_{i'''}\sum_{j=n_4+1}^{P_{i'''}}E_{ji'''} > 0 \tag{4.13}$$

We use the above corollary $m_1 = 0$ and we obtain the Aleph-function of two variables defined by Kumar [8] and Sharma [14], and we get:

Corollary 5.

$$\int_0^1 x^{u-1}\ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right)K(\sqrt{1-x})\,\aleph(z_1x^A, z_2x^B)\mathrm{d}x = \pi a\sum_{n'=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'}n'!}\,\mathrm{a}^{2n'}$$

$$\aleph^{0,n_1+2:m_3,n_3;m_4,n_4}_{P_i+2,Q_i+2,\tau_i;r:P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|l} z_1 & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot & \\ \cdot & \\ z_2 & \qquad [\tau_i(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i},\mathbf{B}_1: \end{array}\right.$$

$$\left.\begin{array}{l}(\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}}\end{array}\right) \tag{4.14}$$

With the following conditions $m_1 = 0$ applied in the corollary 4 and $A, B > 0, |argz_1| < \frac{1}{2}U_3''''\pi, |argz_2| < \frac{1}{2}U_4''''\pi$, U_3'''' and U_4'''' are defined respectively by the relations

$$U_3'''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \tau_i\sum_{j=n_1+1}^{P_i}\alpha_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}\beta_{ji} - \tau_{i''}\sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''} - \tau_{i''}\sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''} > 0 \tag{4.15}$$

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)

https://doi.org/10.31033/ijrasb.8.2.1

and

$$U_4'''' = \sum_{j=1}^{n_1} A_j + \sum_{j=1}^{n_4} E_j + \sum_{j=1}^{m_4} F_j - \tau_i \sum_{j=n_1+1}^{P_i} A_{ji} - \tau_i \sum_{j=m_1+1}^{Q_i} B_{ji} - \tau_{i'} \sum_{j=m_2+1}^{Q_{i'}} B_{ji'}$$

$$-\tau_{i''} \sum_{j=m_4+1}^{Q_{i'''}} F_{ji'''} - \tau_{i'''} \sum_{j=n_4+1}^{P_{i'''}} E_{ji'''} > 0 \tag{4.16}$$

Taking r_i, $r_{i,}'$ r_i'', r_i''' →1 the Aleph-function cited in the corollary 5 becomes the I-function of two variables defined by Sharma and Mishra [15] and we have :

Corollary 6.

$$\int_0^1 x^{u-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x})\, I(z_1 x^A, z_2 x^B) \mathrm{d}x = \pi a \sum_{n'=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'} n'!} \mathrm{a}^{2n'}$$

$$I^{0,n_1+2:m_3,n_3;m_4,n_4}_{P_i+2,Q_i+2;r:P_{i''},Q_{i''};r'':P_{i'''},Q_{i'''};r'''}\left(\begin{array}{c|c} \mathrm{z}_1 & \mathbf{A}_1, (a_j,\alpha_j,A_j)_{1,n_1}, [(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} : \\ \cdot & \\ \cdot & \\ \mathrm{z}_2 & [(b_{ji},\beta_{ji},B_{ji})]_{1,Q_i}, \mathbf{B}_1 : \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{c}_j,\gamma_j)_{1n_3}, [(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}}; (e_j,E_j)_{1n_4}, [(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{d}_j,\delta_j)_{1m_3}, [(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}}; (f_j,F_j)_{1m_4}, [(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right) \tag{4.17}$$

Provided that conditions and notations utilized in the corollary 5, with $\tau_i, \tau_{i''}, \tau_{i'''} \to 1$, and $A, B > 0$, $|arg z_1| < \frac{1}{2} U_3''''' \pi$, $|arg z_2| < \frac{1}{2} U_4''''' \pi$, U_3''''' and U_4''''' are defined respectively by the relations

$$U_3''''' = \sum_{j=1}^{n_1} \alpha_j + \sum_{j=1}^{n_3} \gamma_j + \sum_{j=1}^{m_3} \delta_j - \sum_{j=n_1+1}^{P_i} \alpha_{ji} - \sum_{j=m_1+1}^{Q_i} \beta_{ji} - \sum_{j=m_3+1}^{Q_{i''}} \delta_{ji''} - \sum_{j=n_3+1}^{P_{i''}} \gamma_{ji''} > 0 \tag{4.18}$$

and

$$U_4''''' = \sum_{j=1}^{n_1} A_j + \sum_{j=1}^{n_4} E_j + \sum_{j=1}^{m_4} F_j - \tau_i \sum_{j=n_1+1}^{P_i} A_{ji} - \sum_{j=m_1+1}^{Q_i} B_{ji} - \sum_{j=m_2+1}^{Q_{i'}} B_{ji'}$$

$$- \sum_{j=m_4+1}^{Q_{i'''}} F_{ji'''} - \sum_{j=n_4+1}^{P_{i'''}} E_{ji'''} > 0 \tag{4.19}$$

Now, we consider the situation where r=r’=r’’=r’’’=0 the I-function of two variables reduces to H-function of two variables defined by Gupta and Mittal [7] and we obtain :

Corollary 7.

$$\int_0^1 x^{u-1} \ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right) K(\sqrt{1-x})\, H(z_1 x^A, z_2 x^B) \mathrm{d}x = \pi a \sum_{n'=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'} n'!} \mathrm{a}^{2n'}$$

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)

https://doi.org/10.31033/ijrasb.8.2.1

$$H^{0,n_1+2:m_3,n_3;m_4,n_4}_{P+2,Q+2:P'',Q'':P''',Q'''}\left(\begin{array}{c|c} z_1 & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,P_1}: \ (c_j,\gamma_j)_{1,P_3};(e_j,E_j)_{1,P_4} \\ \cdot & \cdot \qquad\qquad \cdot \\ \cdot & \cdot \qquad\qquad \cdot \\ z_2 & (b_j,\beta_j,B_j)_{1,Q_1},\mathbf{B}_1: \ (d_j,\delta_j)_{1,Q_3};(f_j,F_j)_{1,Q_4} \end{array}\right) \tag{4.20}$$

By respecting the following conditions cited in the corollary 6 with r = r" = r"' =0 and $A,B>0$, $|argz_1|<\frac{1}{2}\mathbf{U}_3\pi, |argz_2|<\frac{1}{2}\mathbf{U}_4\pi$, $\mathbf{U}_3$ and $\mathbf{U}_4$ are defined by

$$\mathbf{U}_3=\sum_{j=1}^{n_1}\alpha_j+\sum_{j=1}^{n_3}\gamma_j+\sum_{j=1}^{m_3}\delta_j-\sum_{j=n_1+1}^{P_1}\alpha_j-\sum_{j=m_1+1}^{Q_1}\beta_j-\sum_{j=m_3+1}^{Q_3}\delta_j-\sum_{j=n_3+1}^{P_3}\gamma_j>0 \tag{4.21}$$

$$\mathbf{U}_4=\sum_{j=1}^{n_1}A_j+\sum_{j=1}^{n_4}E_j+\sum_{j=1}^{m_4}F_j-\sum_{j=n_1+1}^{P_1}A_j-\sum_{j=m_1+1}^{Q_1}B_j-\sum_{j=m_4+1}^{Q_4}F_j-\sum_{j=n_4+1}^{P_4}E_j>0 \tag{4.22}$$

Let $m_1=0;(\alpha_j)_{1,P_1}=(A_j)_{1,P_1}=(\gamma_j)_{1,P_3}=(E_j)_{1,P_4}=(\beta_j)_{1,Q_1}=(B_j)_{1,q_1}=1=(\delta_j)_{1,Q_3}=(F_j)_{1,Q_4}=A=B,$ the H-function of two variables is replaced by the Meijer G-function of two variables defined by Agarwal [1]. Then

Corollary 8.

$$\int_0^1 x^{u-1}\ln\left(\frac{1+a\sqrt{x}}{1-a\sqrt{x}}\right)K(\sqrt{1-x})G(z_1x,z_2x)\mathrm{d}x=\pi a\sum_{n'=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n'}}{\left(\frac{3}{2}\right)_{n'}n'!}a^{2n'}$$

$$G^{0,n_1+2:m_3,n_3;m_4,n_4}_{P+2,Q+2:P'',Q'':P''',Q'''}\left(\begin{array}{c|c} z_1 & A_1',(a_j)_{1,P_1}: \ (c_j)_{1,P_3};(e_j)_{1,P_4} \\ \cdot & \cdot \qquad\qquad \cdot \\ \cdot & \cdot \qquad\qquad \cdot \\ z_2 & (b_j)_{1,Q_1},B_1': \ (d_j)_{1,Q_3};(f_j)_{1,Q_4} \end{array}\right) \tag{4.23}$$

Under the conditions verified by the corollary 7 and the conditions mentionned at the beginning of the corollary 8 and $|argz_1|<\frac{1}{2}U_1\pi, |argz_2|<\frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

$$U_1=\left[n_1+m_3+n_3-\tfrac{1}{2}(p_1+q_1+p_3+q_3)\right] \tag{4.24}$$

and

$$V_1=\left[n_1+m_4+n_4-\tfrac{1}{2}(p_1+q_1+p_4+q_4)\right] \tag{4.25}$$

Remarks

We have the same generalized finite integral with the modified generalized of I-function of two variables defined by Kumari et al. [9], see Singh and Kumar for more details [16] and the special cases, the I-function defined by Saxena [13], the I-function defined by Rathie [12], the Fox's H-function, We have the same generalized multiple finite integrals with the incomplete aleph-function defined by Bansal et al. [3], the incomplete I-function studied by Bansal and Kumar.[2] and the incomplete Fox's H-function given by Bansal et al. [4], the Psi function defined by Pragathi et al. [9].

V. CONCLUSION

The importance of our all the results lies in their manifold generality. First by specializing the various parameters aswell as variable of the generalized modified Aleph-function of two variables, we obtain a large number of results involving remarkably wide variety of useful special functions (or product of such special functions) which are expressible in terms of the Aleph-function of two or one variables, the I-function of two variables or one variable defined by Sharma and Mishra [8] , the H-function of two or one variables , Meijer's G-function of two or one variables and

ISSN: 2349-8889
Volume-8, Issue-2 (March 2021)
www.ijrasb.com
https://doi.org/10.31033/ijrasb.8.2.1

hypergeometric function of two or one variables . Secondly, by specializing the parameters of this unified finite integral, we can get a big number of integrals involving the modified generalized I-functions of two variables and the others functions seen in this document.

REFERENCES

[1] R.P. Agarwal, "An extension of Meijer's G-function," Proc. Nat. Inst. Sci. India Part A, 31 (1965), 536-546.
[2] M.K. Bansal and D. Kumar, On the integral operators pertaining to a family of incomplete I-functions, AIMS Mathematics 5(2) (2020), 1247-1259.
[3] M.K. Bansal, D. Kumar, K.S. Nisar and J. Singh, Certain fractional calculus and integral transform results of incomplete Aleph-functions with applications, Math. Mech; Appli. Sci (Wiley), (2020), 1-13.
[4] M.K. Bansal, D. Kumar, I. Khan, J. Singh and K.S. Nisar, Certain unified integrals associated with product of Mseries and incomplete H-functions, Mathematics, (7) (2019), 1-11.
[5] B.L.J. Braaksma, Asymptotics expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15 (1962-1964), 239-341.
[6] Y.A. Brychkov, Handbook of special functions, Derivatives, Integrals, Series and oher formulas, CRC. Press. Taylor and Francis Group. Boca. Raton, London, New York 2008.
[7] K.C. Gupta, and P.K. Mittal, Integrals involving a generalized function of two variables, (1972), 430-437.
[8] D.Kumar, Generalized fractional differintegral operators of the Aleph-function of two variables, Journal of Chemical, Biological and Physical Sciences, Section C, 6(3) (2016), 1116-1131.
[9] K S. Kumari, T.M. Vasudevan Nambisan and A.K. Rathie, A study of I-functions of two variables, Le matematiche 69(1) (2014), 285-305.
[10] Y. Pragathi Kumar and B. Satyanarayana, A study of Psi-function, Journal of Informatics and mathematical Sciences, Vol. 12 (2) (2020), 159-171.
[11] Y.N. Prasad and S. Prasad, (1979 -1980): Journal of scientific research, Banaras Hindu University, 30 (1), 67 -76.
[12] A.K. Rathie, A new generalization of generalized hypergeometric functions, Le Matematiche, 52 (2) (1997), 297-310.
[13] V.P. Saxena, The I-function, Anamaya Publishers, New Delhi, 2008.
[14] K. Sharma, On the Integral Representation and Applications of the Generalized Function of Two Variables, International Journal of Mathematical Engineering and Science, 3(1) (2014), 1-13.
[15] C.K. Sharma and P.L. Mishra, On the I-function of two variables and its Certain properties, Acta Ciencia Indica, 17 (1991), 1-4.
[16] H. Singh and P. Kumar, Finite integral formulas involving multivariable Mittag-Leffler function and Modified I- function, Int. Jr. of Mathematical sciences and Application, Vol 8(2), (2018), 115-128.
[17] L. J. Slater, Generalized Hypergeometric Functions, Cambridge Univ. Press, Cambridge, 1966.