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Finite Integral Involving the Modified Generalized Aleph-Function of Two Variables Extension and Generalized Extented Hurwitz's Zeta Function

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ABSTRACT

In the present paper, we evaluate the general finite integral invoving the generalized Zeta function and the modified of generalized Aleph-function of two variables.

Keywords- double Mellin-Barnes integrals contour, arcsin function, extension of M-series, Generalized modified Aleph-function of two variables, generalized modified I-function of two variables, generalized modified H-function, of two variables, generalized modified Meijer-function, of two variables Aleph-function of two variables, I-function of two variables, H-function of two variables, Meijer G-function of two variables, extented Zta function.

2010 Mathematics Subject Classification- 33C05, 33C60.

I. INTRODUCTION AND PRELIMINARIES

Recently Aleph-function of two variables has been introduced and studied by Sharma [21], Kumar [9], it's an extension of I-function of two variables defined Sharma and Mishra [22] which is a generalization of the H-function of two variables due to Gupta and Mittal. [8]. On the other hand, Prasad and Prasad [18] have defined the modified generalized H-function of two variables. In this paper, first, we introduce and study the generalized modified Aleph-function of two variables. This function unify the Aleph-function of two variables and the modified of generalized H-function of two variables. Later, we calculate a finite generalized integral involving this function. At the end, we will given several special cases and remarks.

The double Mellin-Barnes integrals contour occurring in this paper will be referred to as the generalized function of two variables throughout our present study and will be defined and represented as follows:

$$\aleph_1(z_1,z_2) = \aleph^{m_1,n_1;m_2,n_2;m_3,n_3;m_4,n_4}_{P_i,Q_i,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|l} \mathrm{z}_1 & (\mathrm{a}_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} : \\ \cdot & \\ \cdot & \\ \mathrm{z}_2 & (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i} : \end{array}\right.$$

$$\left.\begin{array}{l} (\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}} : (\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}} : (\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)x^s y^t \mathrm{d}s\mathrm{d}t, \qquad (1.1)$$

where

$$\phi(s,t) = \frac{\prod_{j=1}^{n_1}\Gamma(1-a_j+\alpha_j s+A_j t)\prod_{j=1}^{m_1}\Gamma(b_j-\beta_j s-B_j t)}{\sum_{i=1}^{r}\tau_i\left[\prod_{j=m_1+1}^{Q_i}\Gamma(1-b_{ji}+\beta_{ji}s+B_{ji}t)\prod_{n_1+1}^{P_i}\Gamma(a_{ji}-\alpha_{ji}s-A_{ji}t)\right]}$$

$$\frac{\prod_{j=1}^{n_2}\Gamma(1-a'_j+\alpha'_js-A'_jt)\prod_{j=1}^{m_2}\Gamma(b'_j-\beta'_js+B'_jt)}{\sum_{i'=1}^{r'}\tau_{i'}\left[\prod_{j=m_2+1}^{Q_{i'}}\Gamma(1-b'_{ji'}+\beta'_{ji'}s-B'_{ji'}t)\prod_{n_2+1}^{P_{i'}}\Gamma(a'_{ji'}-\alpha'_{ji'}s+A'_{ji'}t)\right]} \quad (1.2)$$

$$\theta_1(s)=\frac{\prod_{j=1}^{m_3}\Gamma(d_j-\delta_js)\prod_{j=1}^{n_3}\Gamma(1-c_j+\gamma_js)}{\sum_{i''=1}^{r''}\tau_{i''}\left[\prod_{j=m_3+1}^{Q_{i''}}\Gamma(1-d_{ji''}+\delta_{ji''}s)\prod_{j=n_3+1}^{P_{i''}}\Gamma(c_{ji''}-\gamma_{ji''}s)\right]} \quad (1.3)$$

$$\theta_2(t)=\frac{\prod_{j=1}^{m_4}\Gamma(f_j-F_jt)\prod_{j=1}^{n_4}\Gamma(1-e_j+E_jt)}{\sum_{i'''=1}^{r'''}\tau_{i'''}\left[\prod_{j=m_4+1}^{Q_{i'''}}\Gamma(1-f_{ji'''}+F_{ji'''}t)\prod_{j=n_4+1}^{P_{i'''}}\Gamma(e_{ji'''}-E_{ji'''}t)\right]} \quad (1.4)$$

where z_1 and z_2 (real or complex) are not equal to zero and an empty product is interpreted as unity and the quantities

$P_i, P_{i'}, P_{i''}, P_{i'''}, Q_i, Q_{i'}, Q_{i''}, Q_{i'''}, m_1, m_2, m_3, m_4, n_1, n_2, n_3, n_4$ are non-negative integers such that

$Q_i>0, Q_{i'}>0, Q_{i''}>0, Q_{i'''}>0; \tau_i, \tau_{i'}, \tau_{i''}, \tau_{i'''}>0 (i=1,\cdots,r), (i'=1,\cdots,r'), (i''=1,\cdots,r''), (i'''=1,\cdots,r''')$.

All the $A's, \alpha's, B's, \beta's, A''s, B''s, \alpha''s, \beta''s, \gamma's, \delta's, E's$ and $F's$ are assumed to be positive quantities for standardization purpose ; the definition of generalized Aleph-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the s-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(b_j-\beta_js-B_jt)(j=1,\cdots,m_1)$, $\Gamma(d_j-\delta_js)(j=1,\cdots,m_3)$ and $\Gamma(b'_j-\beta'_js+B'_jt)(j=1,\cdots,m_2)$, are to the right and all the poles of $\Gamma(1-a_j+\alpha_js+A_jt)(j=1,\cdots,n_1)$, $\Gamma(1-c_j+\gamma s)(j=1,\cdots,n_3)$ and $\Gamma(1-a'_j+\alpha'_js-A'_jt)(j=1,\cdots,n_2)$ lie to the left of L_1. The contour L_2 is in the t-plane and runs from $-\omega\infty$ to $+\omega\infty$ with loops, if necessary, to ensure that the poles of $\Gamma(b_j-\beta_js-B_jt)(j=1,\cdots,m_1)$, $\Gamma(f_j-F_jt)(j=1,\cdots,m_4)$ $\Gamma(1-a'_j+\alpha'_js-A'_jt)(j=1,\cdots,n_2)$ are to the right and all the poles of $\Gamma(1-a_j+\alpha_js+A_jt)(j=1,\cdots,n_1)$, $\Gamma(1-e_j+E_jt)(j=1,\cdots,n_3)$ and $\Gamma(b'_j-\beta'_js+B'_jt)(j=1,\cdots,m_2)$ lie to the left of L_2. The poles of the integrand are assumed to be simple.

The function defined by the equation (3.1) is analytic function of z_1 if and z_2 if

$$U_1=\tau_i\sum_{j=1}^{P_i}\alpha_{ji}-\tau_i\sum_{j=1}^{Q_i}\beta_{ji}+\tau_{i'}\sum_{j=1}^{P_{i'}}\alpha_{ji'}-\tau_{i'}\sum_{j=1}^{Q_{i'}}\beta_{ji'}+\tau_{i''}\sum_{j=1}^{P_{i''}}\gamma_{ji''}-\tau_{i''}\sum_{j=1}^{Q_{i''}}\delta_{ji''}<0 \quad (1.5)$$

$$U_2=\tau_i\sum_{j=1}^{P_i}A_{ji}-\tau_i\sum_{j=1}^{Q_i}B_{ji}+\tau_{i'}\sum_{j=1}^{P_{i'}}A_{ji'}-\tau_{i'}\sum_{j=1}^{Q_{i'}}B'_{ji'}+\tau_{i'''}\sum_{j=1}^{P_{i'''}}E_{ji'''}-\tau_{i'''}\sum_{j=1}^{Q_{i'''}}F_{ji'''}<0 \quad (1.6)$$

The integral defined by (2.1) is converges absolutely, if

$$U_3=\sum_{j=1}^{n_1}\alpha_j+\sum_{j=1}^{n_2}\alpha'_j+\sum_{j=1}^{m_1}\beta_j+\sum_{j=1}^{m_2}\beta'_j+\sum_{j=1}^{n_3}\gamma_j+\sum_{j=1}^{m_3}\delta_j-\tau_i\sum_{j=n_1+1}^{P_i}\alpha_{ji}-\tau_i\sum_{j=m_1+1}^{Q_i}\beta_{ji}-\tau_{i'}\sum_{j=m_2+1}^{Q_{i'}}\beta_{ji'}$$

$$-\tau_{i'}\sum_{j=n_2+1}^{P_{i'}}\alpha_{ji'}-\tau_{i''}\sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''}-\tau_{i''}\sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''}>0 \quad (1.7)$$

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$$U_4 = \sum_{j=1}^{n_1} A_j + \sum_{j=1}^{n_2} A'_j + \sum_{j=1}^{m_1} B_j + \sum_{j=1}^{m_2} B'_j + \sum_{j=1}^{n_4} E_j + \sum_{j=1}^{m_4} F_j - \tau_i \sum_{j=n_1+1}^{P_i} A_{ji} - \tau_i \sum_{j=m_1+1}^{Q_i} B_{ji} - \tau_{i'} \sum_{j=m_2+1}^{Q_{i'}} B_{ji'}$$

$$-\tau_{i'} \sum_{j=n_2+1}^{P_{i'}} A_{ji'} - \tau_{i'''} \sum_{j=m_4+1}^{Q_{i'''}} F_{ji'''} - \tau_{i'''} \sum_{j=n_4+1}^{P_{i'''}} E_{ji'''} > 0 \tag{1.8}$$

and

$$|arg z_1| < \frac{\pi}{2} U_3 \text{ and } |arg z_2| < \frac{\pi}{2} U_4 \tag{1.9}$$

We may establish the the asymptotic behavior in the following convenient form, see B.L.J. Braaksma [6]:

$$\aleph(z_1, z_2) = 0(|z_1|^{\alpha_1}, |z_r|^{\alpha_2}), max(|z_1|, |z_2|) \to 0$$

$$\aleph(z_1, z_2) = 0(|z_1|^{\beta_1}, |z_2|^{\beta_2}), min(|z_1|, |z_2|) \to \infty \ :$$

where $\alpha_1 = \min_{1 \leqslant j \leqslant m_3} Re \left[\left(\frac{d_j}{\delta_j} \right) \right]$ and $\alpha_2 = \min_{1 \leqslant j \leqslant m_4} Re \left[\left(\frac{f_j}{F_j} \right) \right]$

where $\beta_1 = \max_{1 \leqslant j \leqslant n_3} Re \left[\left(\frac{c_j - 1}{\gamma_j} \right) \right]$ and $\beta_2 = \max_{1 \leqslant j \leqslant n_4} Re \left[\left(\frac{e_j - 1}{E_j} \right) \right]$

Now, we interest by the v extension of the Mittag-Leffler function. Mittag-Leffler function [11-13] has defined the function named Mittag-Leffler function.

$$\mathrm{E}_\xi(z) = \sum_{k=1}^{\infty} \frac{z^k}{\Gamma(\xi k+1)k!} \tag{1.10}$$

where $z, \xi, \in \mathbb{C}$ and we pose

$$\mathrm{A}_\xi(k) = \frac{1}{\Gamma(\xi k+1)k!} \tag{1.11}$$

Later, Wiman [26,27] defined the following function :

$$\mathrm{E}_{\xi,v}(z) = \sum_{k=1}^{\infty} \frac{z^p}{\Gamma(\xi k+v)k!} \tag{1.12}$$

where $z, \xi, v \in \mathbb{C}, \min\{Re(\xi), Re(v)\} > 0.$

We pose

$$\mathrm{A}_{\xi,v}(p) = \frac{1}{\Gamma(\xi p+v)p!} \tag{1.13}$$

Prabhakar [15] defined and studied the function :

$$\mathrm{E}_{\xi,v}^{\delta}(z) = \sum_{k=1}^{\infty} \frac{(\delta)_p z^k}{\Gamma(\xi k+v)k!} \tag{1.14}$$

Let

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$$\mathrm{A}_{\xi,\upsilon}^{\delta,}(k) = \frac{(\delta)_\rho}{\Gamma(\xi k+\upsilon)k!} \tag{1.15}$$

Recently Prajapati and Shukla [17] studied the functions

$$\mathrm{E}_{\xi,\upsilon}^{\delta,q}(z) = \sum_{k=1}^{\infty} \frac{(\delta)_\rho z^k}{\Gamma(\xi k+\upsilon)k!} \tag{1.16}$$

where $z, \upsilon, \delta, \xi \in \mathbb{C}, \quad \min\{\upsilon, \delta, \xi\} > 0, q \in (0,1) \sqcup \mathbb{N}$

We consider

$$\mathrm{A}_{\xi,\upsilon}^{\delta,q}(k) = \frac{(\delta)_{kq}}{\Gamma(\xi k+\upsilon)k!} \tag{1.17}$$

Provided that : $\mathrm{p} \in \mathbb{R}_0^+, \mathrm{Re}(\xi), Re(\delta), Re(c) > 0$. we will note :

$$\mathrm{A}_{\xi,\upsilon}^{\delta,c}(k;p) = \frac{B_p(\delta+n,c-\delta)}{B(\delta,c-\delta)} \frac{(c)_k}{\Gamma(\xi k+\upsilon)} \frac{1}{k!} \tag{1.19}$$

and

$$\mathrm{B}_p(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1} e^{-\frac{p}{t(1-t)}} \mathrm{d}t \tag{1.20}$$

where $\min\{Re(p), Re(x), Re(y)\} > 0$. Now, we have the generalization of the extended Beta function, see Arshad et al. [2] for more precisions, then :

$$\mathrm{B}_p^{\lambda,\rho}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1} {}_1F_1\left[\lambda; \rho; -\frac{p}{t(1-t)}\right] \mathrm{d}t \tag{1.21}$$

provided that : $\mathrm{p} \in \mathbb{R}^+, \min\{Re(x), Re(y)\} > 0$.

By using the definition of the generalization of the extended Beta function, we defined an extension of the extended Mittag-Leffler function by :

$$\mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(z,p) = \sum_{k=0}^{\infty} \frac{B_p(\gamma+k,c-\gamma)}{B(\gamma,c-\gamma)} \frac{(c)_k}{\Gamma(\alpha k+\beta)} \frac{z^k}{k!} \tag{1.22}$$

Ozarlan and Yilmaz [14] introduced and defined the extended Mittag-Leffler function as follows :

$$\mathrm{E}_{\xi,\upsilon}^{\delta,c}(z) = \sum_{k=0}^{\infty} \frac{B_p(\delta+n,c-\delta)}{B(\delta,c-\delta)} \frac{(c)_k}{\Gamma(\xi k+\upsilon)} \frac{z^k}{k!} \tag{1.18}$$

where $\mathrm{p} \in \mathbb{R}^+, \min\{Re(c), Re(\gamma), Re(\alpha)\} > 0, \rho \in \mathbb{C} \setminus \mathbb{Z}_0^-$

By commodity, we will pose

$$\mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p) = \frac{B_p(\gamma+k,c-\gamma)}{B(\gamma,c-\gamma)} \frac{(c)_k}{\Gamma(\alpha k+\beta)} \frac{1}{k!} \tag{1.23}$$

II. REQUIRED INTEGRAL

In this section, we give a generalized finite integral, see Brychkov ([7], 4.1.1 Eq.17 page 113). We have the result:

Lemma.

$$\int_0^a x^s(a-x)^{s+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{d}x = 2^{-2s-1}\sqrt{\pi}a^{2s+\frac{3}{2}}\frac{\Gamma(2s+2)}{\Gamma(2s+\frac{5}{2})}{}_2F_1\left[-v,2s+2;2s+\frac{5}{2}:-\frac{ab}{2}\right] \tag{2.1}$$

provided that $Re(s) > -1|arg(2+ab)| < \pi$.

III. MAIN INTEGRAL

In this section, we study a generalization of the finite integral involving the modified of generalized Aleph-function of two variables.

Theorem.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\ \aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'}\ \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$\aleph^{m_1,n_1+1;m_2,n_2;m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'';P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|c} z_1\left(\frac{a}{2}\right)^{2C} & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot & \cdot \\ \cdot & \cdot \\ z_2\left(\frac{a}{2}\right)^{2D} & (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}:(\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right) \tag{3.1}$$

where

$$\mathbf{A}_1 = (-1-2u-2Bk-n';C,D);\mathbf{B}_1 = \left(-\frac{3}{2}-2u-2Bk-n',2C,2D\right) \tag{3.2}$$

provided $Re(u) > -1, |arg(2+ab)| < \pi, B,C,D > 0,\ -1 < Re(u+kB) + 2C\min\limits_{1\leqslant j\leqslant m_3} Re\left(\frac{d_j}{\delta_j}\right)$ and

$-1 < Re(u+kB) + 2D\min\limits_{1\leqslant j\leqslant m_4} Re\left(\frac{f_j}{F_j}\right)$ and

where $\mathrm{p} \in \mathbb{R}^+, \min\{Re(c), Re(\gamma), Re(\alpha)\} > 0, \rho \in \mathbb{C}\setminus\mathbb{Z}_0^-$ and $|argz_1| < \frac{1}{2}U_3\pi, |argz_2| < \frac{1}{2}U_4\pi$, U_3 and U_4 are defined respectively by the equations (1.7) and (1.8).

Proof.
To prove the theorem, expressing the extended Mittag-Leffler function in series with the help of (1.22) the modified generalized Aleph-function of two variables in double Mellin-Barnes contour integrals with the help of (1.1) and interchange the order of integrations which is justifiable due to absolute convergence of the integral involved in the process. Now collecting the power of x, we obtain J

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$$\mathrm{J} = \int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^{\upsilon}\mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\ \aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$\sum_{k=0}^{\infty}\mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t$$

$$\int_0^a x^{u+Bl+Cs+Dt}(a-x)^{u+Bl+Cs+Dt+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^{\upsilon}\mathrm{dx\ ds\ dt} \tag{3.3}$$

We use the lemma, this gives :

$$\mathrm{J} = \int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^{\upsilon}\mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-\upsilon)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'}\ \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t\,2^{-2Cs-2Dt}a^{2Cs+2Dt}$$

$$\frac{\Gamma(2u+2Bl+2Cs+2Dt+2)}{\Gamma(2u+2Bl+2Cs+2Dt+\frac{5}{2})}\,{}_4F_3\left(\begin{array}{c}-\upsilon, 2u+2Bl+2Cs+2Dt+2\\ \cdots\\ 2u+2Bl+2Cs+2Dt+\frac{5}{2}\end{array}\middle| -\frac{ab}{2}\right)\mathrm{dx\ ds\ dt} \tag{3.4}$$

We replace the Gauss hypergeometric function by the serie $\sum_{n=0}^{\infty}$, **(see Slater [24], under the hypothesis**, we can interchanged this serie and the $(s,t)-$ integrals, we have :

$$\mathrm{J} = \int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^{\upsilon}\mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-\upsilon)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'}\ \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t\,2^{-2Cs-2Dt}a^{2Cs+2Dt}$$

$$\frac{\Gamma(2u+2Bl+2Cs+2Dt+2)}{\Gamma(2u+2Bl+2Cs+2Dt+\frac{5}{2})}\frac{(2u+2Bl+2Cs+2Dt+2)_{n'}}{(2u+2Bl+2Cs+2Dt+\frac{5}{2})_{n'}}\ \mathrm{ds\ dt} \tag{3.5}$$

Now we appel the relation $\Gamma(a)(a)_n = \Gamma(a+n)$, this gives :

$$\mathrm{J} = \int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^{\upsilon}\mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-\upsilon)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'}\ \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi(s,t)\theta_1(s)\theta_2(t)z_1^s z_2^t\ 2^{-2Cs-2Dt}a^{2Cs+2Dt}2^{2At}a^{-2At}\frac{\Gamma\left(2u+2Bl+2Cs+2Dt+2+n'\right)}{\Gamma(2u+2Bl+2Cs+2Dt+\frac{5}{2}+n')}\mathrm{ds\ dtdt} \tag{3.6}$$

We interpret these double integrals contour of the modified generalized Aleph-function of two variables with the help of (1.1), we obtain the desired result.

In the following section, we consider the extended Mittag-Leffler function defined by Arshad et al. [2] and special

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cases of the modified of the generalized Aleph-function of two variables.

IV. SPECIAL CASES

Let $\tau_i, \tau_{i'}, \tau_{i''}, \tau_{i'''}$ -> 1 the generalized modified Aleph-function of two variables reduces to modified of generalized I-function of two variables, this gives

Corollary 1.

$$\int_0^a x^u (a-x)^{u+\frac{1}{2}} \left[1 + b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B, p) \;\; I(z_1 x^C (a-x)^C, z_2 x^D (a-x)^D) \mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi} a^{2u+\frac{3}{2}} \sum_{k,n'=0}^{\infty} \frac{(-v)_{n'}}{n'!} \left(-\frac{ab}{2}\right)^{n'} \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\, \mathrm{Z}^k \, 2^{-2Bk} a^{2Bk}$$

$$I_{P_i,Q_i;r;P_{i'},Q_{i'};r';P_{i''},Q_{i''};r'':P_{i'''},Q_{i'''};r'''}^{m_1,n_1;m_2,n_2:m_3,n_3:m_4,n_4}\left(\begin{matrix} z_1\left(\frac{a}{2}\right)^{2C} \\ \cdot \\ \cdot \\ z_2\left(\frac{a}{2}\right)^{2D} \end{matrix} \middle| \begin{matrix} \mathbf{A}_1, (a_j,\alpha_j,A_j)_{1,n_1}, [(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i} : \\ \cdot \\ \cdot \\ (\mathrm{b}_j,\beta_j,B_j)_{1,m_1}, [(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i}, \mathbf{B}_1 : \end{matrix}\right.$$

$$\left.\begin{matrix} (\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2}, [(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}} : (\mathrm{c}_j,\gamma_j)_{1n_3}, [(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}} ; (e_j,E_j)_{1n_4}, [(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2}, [(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}} \; (\mathrm{d}_j,\delta_j)_{1m_3}, [(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}} ; (f_j,F_j)_{1m_4}, [(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{matrix}\right) \quad (4.1)$$

under the same notations and conditions that theorem with $\tau_i, \tau_{i'}, \tau_{i''}, \tau_{i'''} \to 1$, and $|arg z_1| < \frac{1}{2}U'_3\pi, |arg z_2| < \frac{1}{2}U'_4\pi$, U'_3 and U'_4 are defined by

$$U'_3 = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_2}\alpha'_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{m_2}\beta'_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \sum_{j=n_1+1}^{P_i}\alpha_{ji} - \sum_{j=m_1+1}^{Q_i}\beta_{ji} - \sum_{j=m_2+1}^{Q_{i'}}\beta_{ji'}$$

$$- \sum_{j=n_2+1}^{P_{i'}}\alpha_{ji'} - \sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''} - \sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''} > 0 \quad (4.2)$$

$$U'_4 = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{n_2}A'_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{m_2}B'_j + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \sum_{j=n_1+1}^{P_i}A_{ji} - \sum_{j=m_1+1}^{Q_i}B_{ji} - \sum_{j=m_2+1}^{Q_{i'}}B_{ji'}$$

$$- \sum_{j=n_2+1}^{P_{i'}}A_{ji'} - \sum_{j=m_4+1}^{Q_{i'''}}F_{ji'''} - \sum_{j=n_4+1}^{P_{i'''}}E_{ji'''} > 0 \quad (4.3)$$

We consider the above corollary and we suppose r = r' = r" = r"' =1, we obtain the generalized modified H-function defined by Prasad and Prasad [14] and the following integral.

Corollary 2.

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$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\ \ H(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'} \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$H_{P+1,Q+1:P',Q':P'',Q'':P''',Q'''}^{m_1,n_1+1;m_2,n_2:m_3,n_3;m_4,n_4}\left(\begin{array}{c|c} z_1\left(\frac{a}{2}\right)^{2C} & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,P_1}:\ (a'_j,\alpha'_j,A_j)_{1,P_2}:(c_j,\gamma_j)_{1,P_3};(e_j,E_j)_{1,P_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ z_2\left(\frac{a}{2}\right)^{2D} & (b_j,\beta_j,B_j)_{1,Q_1},\mathbf{B}_1:\ (b'_j,\beta'_j,B_j)_{1,Q_2}:(d_j,\delta_j)_{1,Q_3};(f_j,F_j)_{1,Q_4}\end{array}\right) \quad (4.4)$$

by respecting the conditions cited in the corollary 1 with r =r' =r" =r"' =0.

Taking $(\alpha_j)_{1,P_1}=(A_j)_{1,P_1}=(\alpha'_j)_{1,P_2}=(A'_j)_{1,P_2}=(\gamma_j)_{1,P_3}=(E_j)_{1,P_4}=\ (\beta_j)_{1,Q_1}=(B_j)_{1,q_1}=(\beta'_j)_{1,Q_2}=1=$ C =D = $(B'_j)_{1,Q_2}=(\delta_j)_{1,Q_3}=(F_j)_{1,Q_4}$, the modified generalized H-function of two variables is replaced by the generalized modified of Meijer G-function of two variables defined by Agarwal [1], we have

Corollary 3.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\arcsin^2\left(b\sqrt[4]{x(a-x)}\right)\mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\ \ G(z_1x(a-x), z_2x(a-x))\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'} \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$G_{P+1,Q+1:P',Q':P'',Q'':P''',Q'''}^{m_1,n_1+1;m_2,n_2:m_3,n_3;m_4,n_4}\left(\begin{array}{c|c} z_1 & A'_1,(a_j)_{1,P_1}:\ (a'_j,\alpha'_j,A_j)_{1,P_2}:(c_j,\gamma_j)_{1,P_3};(e_j,E_j)_{1,P_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ z_2 & (b_j)_{1,Q_1},B'_1:\ (b'_j,\beta'_j,B_j)_{1,Q_2}:(d_j,\delta_j)_{1,Q_3};(f_j,F_j)_{1,Q_4}\end{array}\right) \quad (4.5)$$

under the conditions verified by the corollary 2 and the conditions mentionned at the beginning of this corollary where

$$A'_1=(-1-2u-2Bk-n');\ B'_1=\left(-\tfrac{3}{2}-2u-2Bk-n'\right) \quad (4.6)$$

and $|argz_1|<\frac{1}{2}U_1\pi, |argz_2|<\frac{1}{2}V_1\pi$, U_1, V_1 are defined by the following formulas :

$$U_1=\left[m_1+n_1+m_2+n_2+m_3+n_3-\tfrac{1}{2}(p_1+q_1+p_2+q_2+p_3+q_3)\right] \quad (4.7)$$

and

$$V_1=\left[m_1+n_1+m_2+n_2+m_4+n_4-\tfrac{1}{2}(p_1+q_1+p_2+q_2+p_4+q_4)\right] \quad (4.8)$$

Let $m_2=n_2=P_{i'}=Q_{i'}=0$, the modified of generalized Aleph-function of two variables reduces to generalized Aleph-function of two variables, this gives:

Corollary 4.

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$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\ \aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'}\mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$\aleph_{P_i+1,Q_i+1,\tau_i;r:P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}^{m_1,n_1+1:m_3,n_3;m_4,n_4}\left(\begin{array}{c} z_1\left(\frac{a}{2}\right)^{2C} \\ \cdot \\ \cdot \\ z_2\left(\frac{a}{2}\right)^{2D}\end{array}\left|\begin{array}{c}\mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot \\ \cdot \\ (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1:\end{array}\right.\right.$$

$$\left.\begin{array}{c}(\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}}\end{array}\right) \tag{4.9}$$

By respecting the conditions mentionned by the theorem where $m_2 = n_2 = P_{i'} = Q_{i'} = 0$, and the following conditions : $|argz_1| < \frac{1}{2}U_3'''\pi$, $|argz_2| < \frac{1}{2}U_4'''\pi$, U_3''' and U_4''' are defined respectively by the relations :

$$U_3''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \tau_i\sum_{j=n_1+1}^{P_i}\alpha_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}\beta_{ji} - \tau_{i''}\sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''} - \tau_{i''}\sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''} > 0 \tag{4.10}$$

and

$$U_4''' = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \tau_i\sum_{j=n_1+1}^{P_i}A_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}B_{ji} - \tau_{i'}\sum_{j=m_2+1}^{Q_{i'}}B_{ji'}$$

$$-\tau_{i''}\sum_{j=m_4+1}^{Q_{i'''}}F_{ji'''} - \tau_{i'''}\sum_{j=n_4+1}^{P_{i'''}}E_{ji'''} > 0 \tag{4.11}$$

We consider the above corollary with $\mathrm{m}_1 = 0$, we obtain the Aleph-function of two variables defined by Kumar [9] and Sharma [21] , this gives :

Corollary 5.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\ \aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'}\mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$\aleph_{P_i+1,Q_i+1;r:P_{i''},Q_{i''};r'':P_{i'''},Q_{i'''};r'''}^{0,n_1+1:m_3,n_3;m_4,n_4}\left(\begin{array}{c} z_1\left(\frac{a}{2}\right)^{2C} \\ \cdot \\ \cdot \\ z_2\left(\frac{a}{2}\right)^{2D}\end{array}\left|\begin{array}{c}\mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot \\ \cdot \\ (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1:\end{array}\right.\right.$$

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$$\left.\begin{matrix}(c_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}}\\ \cdot \\ \cdot \\ (d_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}}\end{matrix}\right) \quad (4.12)$$

with the conditions verified by the theorem where $m_1 = m_2 = n_2 = P_{i'} = Q_{i'} = 0$, and the following conditions : $|argz_1| < \frac{1}{2}U_3'''\pi, |argz_2| < \frac{1}{2}U_4'''\pi$, U_3''' and U_4''' are defined respectively by the relations

$$U_3''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \tau_i\sum_{j=n_1+1}^{P_i}\alpha_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}\beta_{ji} - \tau_{i''}\sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''} - \tau_{i''}\sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''} > 0 \quad (4.13)$$

and

$$U_4''' = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{m_1}B_j + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \tau_i\sum_{j=n_1+1}^{P_i}A_{ji} - \tau_i\sum_{j=m_1+1}^{Q_i}B_{ji} - \tau_{i'}\sum_{j=m_2+1}^{Q_{i'}}B_{ji'}$$

$$-\tau_{i''}\sum_{j=m_4+1}^{Q_{i'''}}F_{ji'''} - \tau_{i'''}\sum_{j=n_4+1}^{P_{i'''}}E_{ji'''} > 0 \quad (4.14)$$

Let $\tau_i, \tau_{i''}, \tau_{i'''} \to 1$, the Aleph-function of two variables reduces to I-function of two variables defined by Sharma and Mishra [22] and we have :

Corollary 6.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B,p)\ \ I(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'}\mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\,\mathrm{Z}^k\,2^{-2Bk}a^{2Bk}$$

$$I^{0,n_1+1:m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1;r:P_{i''},Q_{i''};r'':P_{i'''},Q_{i'''};r'''}\left(\begin{matrix}z_1\left(\frac{a}{2}\right)^{2C}\\ \cdot \\ \cdot \\ z_2\left(\frac{a}{2}\right)^{2D}\end{matrix}\ \middle|\ \begin{matrix}\mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}:\\ \cdot \\ \cdot \\ (b_j,\beta_j,B_j)_{1,m_1},[(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j,\gamma_j)_{1n_3},[(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}}\\ \cdot \\ \cdot \\ (d_j,\delta_j)_{1m_3},[(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}}\end{matrix}\right) \quad (4.15)$$

Under the conditions verified by the corollary 5 and $\tau_i, \tau_{i''}, \tau_{i'''} \to 1$, where

$|argz_1| < \frac{1}{2}U_3''''' \pi, |argz_2| < \frac{1}{2}U_4'''''\pi$, U_3''''' and U_4''''' are defined respectively by the relations

$$U_3''''' = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{n_3}\gamma_j + \sum_{j=1}^{m_3}\delta_j - \sum_{j=n_1+1}^{P_i}\alpha_{ji} - \sum_{j=m_1+1}^{Q_i}\beta_{ji} - \sum_{j=m_3+1}^{Q_{i''}}\delta_{ji''} - \sum_{j=n_3+1}^{P_{i''}}\gamma_{ji''} > 0 \quad (4.16)$$

and

$$U_4''''' = \sum_{j=1}^{n_1}A_j + \sum_{j=1}^{n_4}E_j + \sum_{j=1}^{m_4}F_j - \tau_i\sum_{j=n_1+1}^{P_i}A_{ji} - \sum_{j=m_1+1}^{Q_i}B_{ji} - \sum_{j=m_2+1}^{Q_{i'}}B_{ji'}$$

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$$- \sum_{j=m_4+1}^{Q_{i'''}} F_{ji'''} - \sum_{j=n_4+1}^{P_{i'''}} E_{ji'''} > 0 \tag{4.17}$$

Taking r = r" = r"' =0, the I-function of two variables reduces to H-function of two variables defined by Gupta and Mittal [8] and we obtain :

Corollary 7.

$$\int_0^a x^u (a-x)^{u+\frac{1}{2}} \left[1 + b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B, p) \;\; H(z_1 x^C (a-x)^C, z_2 x^D (a-x)^D) \mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi} a^{2u+\frac{3}{2}} \sum_{k,n'=0}^{\infty} \frac{(-v)_{n'}}{n'!} \left(-\frac{ab}{2}\right)^{n'} \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\, \mathrm{Z}^k\, 2^{-2Bk} a^{2Bk}$$

$$H_{P+1,Q+1:P'',Q'';P''',Q'''}^{0,n_1+1:m_3,n_3;m_4,n_4}\left(\begin{array}{c|c} z_1 & \mathbf{A}_1, (a_j, \alpha_j, A_j)_{1,P_1} : \; (\mathrm{c}_j, \gamma_j)_{1,P_3}; (e_j, E_j)_{1,P_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ z_2 & (\mathrm{b}_j, \beta_j, B_j)_{1,Q_1}, \mathbf{B}_1 : \; (\mathrm{d}_j, \delta_j)_{1,Q_3}; (f_j, F_j)_{1,Q_4} \end{array}\right) \tag{4.18}$$

By considering the conditions and notations verified by the corollary 6 and r = r" = r"' =0, where $|argz_1| < \frac{1}{2}\mathbf{U}_3\pi, |argz_2| < \frac{1}{2}\mathbf{U}_4\pi$, $\mathbf{U}_3$ and $\mathbf{U}_4$ are defined by

$$\mathbf{U}_3 = \sum_{j=1}^{n_1} \alpha_j + \sum_{j=1}^{n_3} \gamma_j + \sum_{j=1}^{m_3} \delta_j - \sum_{j=n_1+1}^{P_1} \alpha_j - \sum_{j=m_1+1}^{Q_1} \beta_j - \sum_{j=m_3+1}^{Q_3} \delta_j - \sum_{j=n_3+1}^{P_3} \gamma_j > 0 \tag{4.19}$$

$$\mathbf{U}_4 = \sum_{j=1}^{n_1} A_j + \sum_{j=1}^{n_4} E_j + \sum_{j=1}^{m_4} F_j - \sum_{j=n_1+1}^{P_1} A_j - \sum_{j=m_1+1}^{Q_1} B_j - \sum_{j=m_4+1}^{Q_4} F_j - \sum_{j=n_4+1}^{P_4} E_j > 0 \tag{4.20}$$

Let $\mathrm{m}_1 = 0; (\alpha_j)_{1,P_1} = (A_j)_{1,P_1} = (\gamma_j)_{1,P_3} = (E_j)_{1,P_4} = (\beta_j)_{1,Q_1} = (B_j)_{1,q_1} = 1 = (\delta_j)_{1,Q_3} = (F_j)_{1,Q_4} = C = D$ the H-function of two variables is replaced by the Meijer G-function of two variables defined by Agarwal [1]. Then

Corollary 8.

$$\int_0^a x^u (a-x)^{u+\frac{1}{2}} \left[1 + b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(zx^B(a-x)^B, p) \;\; G(z_1 x(a-x), z_2 x(a-x)) \mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi} a^{2u+\frac{3}{2}} \sum_{k,n'=0}^{\infty} \frac{(-v)_{n'}}{n'!} \left(-\frac{ab}{2}\right)^{n'} \mathrm{A}_{\alpha,\beta}^{\gamma,c;\lambda,\rho}(k;p)\, \mathrm{Z}^k\, 2^{-2Bk} a^{2Bk}$$

$$G_{P+1,Q+1:P'',Q'';P''',Q'''}^{0,n_1+1:m_3,n_3;m_4,n_4}\left(\begin{array}{c|c} z_1 & \mathrm{A}'_1, (a_j)_{1,P_1} : \; (\mathrm{c}_j, \gamma_j)_{1,P_3}; (e_j, E_j)_{1,P_4} \\ \cdot & \cdot \\ \cdot & \cdot \\ z_2 & (\mathrm{b}_j)_{1,Q_1}, B'_1 : \; (\mathrm{d}_j, \delta_j)_{1,Q_3}; (f_j, F_j)_{1,Q_4} \end{array}\right) \tag{4.21}$$

Verifying the conditions of the corollary 7 and the following conditions stated at the beginning of this corollary, $\mathrm{m}_1 = 0; (\alpha_j)_{1,P_1} = (A_j)_{1,P_1} = (\gamma_j)_{1,P_3} = (E_j)_{1,P_4} = (\beta_j)_{1,Q_1} = (B_j)_{1,q_1} = 1 = (\delta_j)_{1,Q_3} = (F_j)_{1,Q_4}$ and

$|argz_1| < \frac{1}{2}U_1\pi, |argz_2| < \frac{1}{2}V_1\pi$, U_1 and V_1 are defined by the following formulas :

$$\mathrm{U}_1 = \left[n_1 + m_3 + n_3 - \tfrac{1}{2}(p_1 + q_1 + p_3 + q_3)\right] \tag{4.22}$$

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and

$$V_1 = \left[n_1 + m_4 + n_4 - \tfrac{1}{2}(p_1 + q_1 + p_4 + q_4)\right] \tag{4.23}$$

Now, we involve the special cases of the extented of Zeta function and we utilize the modified of generalized Alephfunction of two variables defined in the beginning. We will use the same notation that the section 1.

V. PARTICULAR CASES OF THE EXTENTED ZETA FUNCTION

We consider the function defined by Ozarlan and Yilmaz [14], this gives:

Corollary 9.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\delta,c}(zx^B(a-x)^B,p)\ \aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'} 2^{-2Bk}a^{2Bk}\ \mathrm{A}_{\alpha,\beta}^{\delta,c}(k;p)$$

$$\aleph^{m_1,n_1+1;m_2,n_2;m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c} z_1\left(\frac{a}{2}\right)^{2C} \\ \vdots \\ z_2\left(\frac{a}{2}\right)^{2D}\end{array}\middle|\begin{array}{l}\mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \vdots \\ (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1:\end{array}\right.$$

$$\left.\begin{array}{l}(\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(\mathrm{e}_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \vdots \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}:(\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}}\end{array}\right) \tag{5.1}$$

where the conditions and notations verified by the theorem and $\mathrm{p}\in\mathbb{R}_0^+, \mathrm{Re}(\xi), Re(\delta), Re(c)>0$.

Let the function defined Recently by Prajapati and Shukla [17].

Corollary 10.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}^{\delta,c}(Zx^B(a-x)^B)\aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'} 2^{-2Bk}a^{2Bk}\ \mathrm{Z}^k\ \mathrm{A}_{\alpha,\beta}^{\delta,c}(k)$$

$$\aleph^{m_1,n_1+1;m_2,n_2;m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c} z_1\left(\frac{a}{2}\right)^{2C} \\ \vdots \\ z_2\left(\frac{a}{2}\right)^{2D}\end{array}\middle|\begin{array}{l}\mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \vdots \\ (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1:\end{array}\right.$$

$$\left.\begin{array}{l}(\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(\mathrm{e}_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \vdots \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}:(\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}}\end{array}\right) \tag{5.2}$$

under the conditions and notations verified by the theorem and $z,\ v,\delta,\xi\in\mathbb{C},\ \min\{v,\delta,\xi\}>0, q\in(0,1)\sqcup\mathbb{N}$

Corollary 11.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}^{\delta}_{\alpha,\beta}(Zx^B(a-x)^B)\aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'} 2^{-2Bk}a^{2Bk}\ \mathrm{Z}^k\ \mathrm{A}^{\delta}_{\alpha,\beta}(k)$$

$$\aleph^{m_1,n_1+1;m_2,n_2;m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|c} z_1\left(\frac{a}{2}\right)^{2C} & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \vdots & \vdots \\ z_2\left(\frac{a}{2}\right)^{2D} & (b_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1: \end{array}\right.$$

$$\left.\begin{array}{c} (a'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (c_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \vdots \\ (b'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}:(d_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right) \quad (5.3)$$

With the conditions and notations verified by the theorem and $z, v, \delta, \xi \in \mathbb{C}, \min\{Re(\xi), Re(v)\} > 0$.

Considering the function defined by Wiman [26,27] , we get

Corollary 12.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_{\alpha,\beta}(Zx^B(a-x)^B)\ \aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'} 2^{-2Bk}a^{2Bk}\ \mathrm{Z}^k\ \mathrm{A}_{\alpha,\beta}(k)$$

$$\aleph^{m_1,n_1+1;m_2,n_2;m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'':P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|c} z_1\left(\frac{a}{2}\right)^{2C} & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \vdots & \vdots \\ z_2\left(\frac{a}{2}\right)^{2D} & (b_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1: \end{array}\right.$$

$$\left.\begin{array}{c} (a'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (c_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \vdots \\ (b'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}:(d_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right) \quad (5.4)$$

by respecting the conditions and notation of the theorem and $z, \xi, v \in \mathbb{C}, \min\{Re(\xi), Re(v)\} > 0$.

Now, we use the function defined by Mittag-Leffler function [11-13]

Corollary 13.

$$\int_0^a x^u(a-x)^{u+\frac{1}{2}}\left[1+b\sqrt{x(a-x)}\right]^v \mathrm{E}_\xi(Zx^B(a-x)^B)\aleph(z_1x^C(a-x)^C, z_2x^D(a-x)^D)\mathrm{d}x =$$

$$= 2^{-2u-1}\sqrt{\pi}a^{2u+\frac{3}{2}}\sum_{k,n'=0}^{\infty}\frac{(-v)_{n'}}{n'!}\left(-\frac{ab}{2}\right)^{n'} 2^{-2Bk}a^{2Bk}\ \mathrm{Z}^k\,\mathrm{A}_\xi(k)$$

$$\aleph^{m_1,n_1+1;m_2,n_2;m_3,n_3;m_4,n_4}_{P_i+1,Q_i+1,\tau_i;r;P_{i'},Q_{i'},\tau_{i'};r';P_{i''},Q_{i''},\tau_{i''};r'';P_{i'''},Q_{i'''},\tau_{i'''};r'''}\left(\begin{array}{c|c} z_1\left(\frac{a}{2}\right)^{2C} & \mathbf{A}_1,(a_j,\alpha_j,A_j)_{1,n_1},[\tau_i(a_{ji},\alpha_{ji},A_{ji})]_{n_1+1,P_i}: \\ \cdot & \cdot \\ \cdot & \cdot \\ z_2\left(\frac{a}{2}\right)^{2D} & (\mathrm{b}_j,\beta_j,B_j)_{1,m_1},[\tau_i(b_{ji},\beta_{ji},B_{ji})]_{m_1+1,Q_i},\mathbf{B}_1: \end{array}\right.$$

$$\left.\begin{array}{c} (\mathrm{a}'_j,\alpha'_j,A_j)_{1,n_2},[\tau_{i'}(a_{ji'},\alpha_{ji'},A_{ji'})]_{n_2+1,P_{i'}}:\ (\mathrm{c}_j,\gamma_j)_{1n_3},[\tau_{i''}(c_{ji''},\gamma_{ji''})]_{n_3+1;P_{i''}};(e_j,E_j)_{1n_4},[\tau_{i'''}(e_{ji'''},\gamma_{ji'''})]_{n_4+1;P_{i'''}} \\ \cdot \\ \cdot \\ (\mathrm{b}'_j,\beta'_j,B_j)_{1,m_2},[\tau_{i'}(b_{ji'},\beta_{ji'},B_{ji'})]_{m_2+1,Q_{i'}}:(\mathrm{d}_j,\delta_j)_{1m_3},[\tau_{i''}(d_{ji''},\delta_{ji''})]_{m_3+1;Q_{i''}};(f_j,F_j)_{1m_4},[\tau_{i'''}(f_{ji'''},F_{ji'''})]_{m_4+1;Q_{i'''}} \end{array}\right) \quad (5.5)$$

By mentionning the conditions and notations of the theorem and $z, \xi, \in \mathbb{C}$.

Remarks

We have the same generalized finite integral with the modified generalized of I-function of two variables defined by Kumari et al. [10], see Singh and Kumar for more details [23] and the special cases, the I-function defined by Saxena [20], the I-function defined by Rathie [19], the Fox's H-function, We have the same generalized multiple finite integrals with the incomplete aleph-function defined by Bansal et al. [4], the incomplete I-function studied by Bansal and Kumar. [3] and the incomplete Fox's H-function given by Bansal et al. [5], the Psi function defined by Pragathi et al. [16]. We have the same generalized finite integrals involving the extension of Mittag-Leffler function with the alephfunction defined by Sudland [25].

VI. CONCLUSION

The importance of our all the results lies in their manifold generality. First by specializing the various parameters as well as variable of the generalized modified Aleph-function of two variables, we obtain a large number of results involving remarkably wide variety of useful special functions (or product of such special functions) which are expressible in terms of the Aleph-function of two or one variables, the I-function of two variables or one variable defined by Sharma and Mishra [22] , the H-function of two or one variables, Meijer's G-function of two or one variables and hypergeometric function of two or one variables.

Secondly, by specializing the parameters of this unified finite integral, we can get a large number of integrals involving the modified generalized I-functions of two variables and the others functions seen in this document.

Thirdly, by specializing the parameters of the variable of the extented Zeta function, we get a big number of known and news finite integrals.

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