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A Fuzzy Production Inventory Model for Deteriorating Items with Shortages

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ABSTRACT

In this paper we have developed a supply chain production inventory model for deteriorating items with shortage under Fuzzy environment. The formulae for the optimal average system cost, stock level, backlog level and production cycle time are derived when the deterioration rate is very small. In reality it is seen that we cannot define all parameters precisely due to imprecision or uncertainty in the environment. So, we have defined the inventory parameter deterioration rate as triangular fuzzy numbers. The signed distance method and graded mean integration method have been used for defuzzification. Numerical examples are taken to illustrate the procedure of finding the optimal total inventory cost, stock level and backlog level. Sensitivity analysis is carried out to demonstrate the effects of changing parameter values on the optimal solution of the system.

Keywords- Deterioration, shortage, triangular fuzzy numbers (symmetric), defuzzification, signed distance method, graded mean integration method.

I. INTRODUCTION

The most important and difficult role that inventory plays in Supply chain is that of facilitating the balancing of demand and supply. To effectively manage the forward and reverse flows in the supply chain, firms have to deal with upstream supplier exchanges and downstream customer demands. Uncertainty is another key issue to deal with in order to define effective Supply Chain inventory policies. Demand, Supply (e.g. lead time), various relevant cost, backorder costs, deterioration rate etc. are usually uncertain. To solve these types of practical problems we use the Fuzzy Set Theory. Bellman and Zadeh (1970) (1) first studied fuzzy set theory to solve decision making problem. Then, Dubois and Prade (1978) (2) introduced some operations on fuzzy number. Thereafter, Park (1987) (3) developed fuzzy set theoretical interpretation of EOQ. Several researchers like K. Wu and J. S. Yao (2003) (4), X. Wang and R. Zhao (2007) (5), Hu Jinsong et al. (2010) (6), K. Jaggi et al. (2013) (16), J. S. Yao and J. Chiang (2003) (17), X. Wang et al. (2007) (18), C. Kao and W. K. Hsu (2002) (19), P. Dutta et al. (2005) (20), A. Roy and G. P. Samanta (2009) (15) have developed different types of inventory model under Fuzzy environment. In this area, a lot of research papers have been published by several researchers viz., Bera, Bhunia, and Maiti (2013) (21), He, Wei, and Fuyuan (2013) (22), Dutta and Kumar (2015) (23), Mishra et al. (2015) (24) etc. S. Priyan, P. Manivannan (2017) (25) developed an optimal inventory modelling of supply chain system involving quality inspection errors in Fuzzy situation.

In recent years, the control and maintenance of production inventories of deteriorating items with shortages have attracted much attention in inventory analysis because most physical goods deteriorate over time. The effect of deterioration is very important in inventory system. Deterioration is defined as decay or damage such that the item cannot be used for its original purpose. Food items, drugs, pharmaceuticals, radioactive substances are examples of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system. Various researchers have investigated these issues over time. A. Roy and G. P. Samanta (2009) (9) developed an inventory model without backorder for deteriorating items under fuzzy environment. K. Jaggi et al. (2013) (10) studied an inventory model for deteriorating items with time-varying demand and shortages in uncertainty. N. K. Duari and T Chakrabarti (2014) (12) developed an order level EOQ model for deteriorating items in a single warehouse system with price dependent demand and shortages.

Another class of inventory models has been developed with time dependent parameters. S. Shee and T. Chakrabarti (2020) (13) have been developed a two-echelon supply chain model for deteriorating items with time varying holding cost involving lead time as a decision variable where all inventory costs are uncertain. D. Dutta and P. Kumar (2015) (8) analyzed a partial backlogging inventory model for deteriorating items with time-varying demand and holding cost.

S. Shee and T. Chakrabarti (2020) (14) introduced an inventory model for deteriorating items in a supply chain system with Time dependent demand Rate under fuzzy environment. S. Saha and T. Chakrabarti (2016) (11) developed a buyer vendor EOQ model with time varying holding cost involving lead

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time as a decision variable under an integrated Supply chain system.

In the present paper we have developed a continuous production control inventory model for deteriorating items with shortages under fuzzy environment. It is assumed that the demand rate and production rate are constants. The main focus is on the structural behavior of the system. The convexity of the cost function is established to ensure the existence of a unique optimal solution. The optimum inventory level is proved to be a decreasing function of the deterioration rate where the deterioration rate is taken as very small and the cycle time is taken as constant. The formulae for the optimal average cost, stock level, backlog level is derived when the deterioration rate is very small. Here deterioration rate is taken as triangular fuzzy numbers and demand rate is constant. Later on, the fuzzy total cost is defuzzified by using signed distance method and graded mead integration method. Numerical examples are taken and the sensitivity analysis is carried out to demonstrate the effects of changing parameter values on the optimal solution of the system. The problem is solved by using LINGO 17.0 software.

II. NOTATION AND ASSUMPTIONS

The following notations and assumptions are used for developing the model.

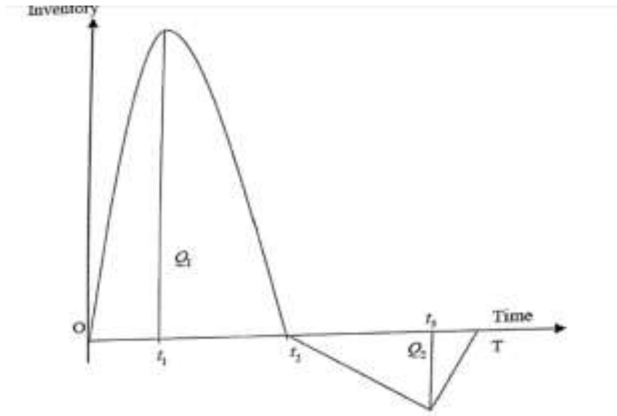
(a) a = the constant demand rate.
(b) p (> a) = the constant production rate.
(c) θ= the deterioration rate (0 < 8 < 1)
(d) $\tilde{\theta}$ = the Fuzzy deterioration rates
(e) T = the cycle length
(f) C_1 =the holding cost per unit per unit time.
(g) C_2 =the shortage cost per unit per unit time.
(h) C_3 = the cost of a deteriorated unit. (C_1, C_2 and C_3 are known constants).
(i) TC = the total inventory cost or the average system cost.
(j) $\tilde{\mathfrak{C}}$ = the fuzzified value of TC

TC_s = DE fuzzified value of $\tilde{\mathfrak{C}}$ when signed distance method of defuzzification is used

(k) TC_G = DE fuzzified value of $\tilde{\mathfrak{C}}$ when Graded mean integration method of defuzzification is used
(l) Replenishment is instantaneous and lead time is zero.
(m) T is the fixed duration of a production cycle.
(n) Shortages are allowed and backlogged.
(o) It is assumed that no repair or placement of the deteriorated items takes place during a given cycle.

Here we assume that the production starts at time t = 0 and stops at time t = t_1. During [0, t_1], the production rate is p and the demand rate is a (< p). The stock attains a level Q_1 at time t = t_1. During [t_1, t_2], the inventory level gradually decreases mainly to meet demands and partly for deterioration. The stock falls to the zero level at time t = t_2. Now shortages occur and accumulate to the level Q_2at time t = t_3. The production starts again at a rate p at t = $t_3$and the backlog is cleared at time t = T when the stock is again zero. The cycle then repeats itself after time T.

This model is represented by the following diagram:



III. MATHEMATICAL MODEL

Let Q(t) be the on-hand inventory at time t(0 ≤ t ≤ T). Then the differential equations governing the instantaneous state of Q (t) at any time t are given by:

$\frac{dQ(t)}{dt}$ + θ Q(t) = (p-a), 0 ≤ t ≤ t_1 ------------------------ (1)

$\frac{dQ(t)}{dt}$ + θ Q(t) = -a, t_1 ≤ t ≤ t_2 ---------------------------(2)

$\frac{dQ(t)}{dt}$ = -a, t_2 ≤ t ≤ t_3 -------------------------------------(3)

$\frac{dQ(t)}{dt}$ = p - a, t_3 ≤ t ≤ T ----------------------------------(4)

The boundary conditions are:

Q(0)=0, Q(t_1)=Q_1, Q(t_2)=0, Q(t_3)= -Q_2 , Q(T)=0 -----(5)

The solution of equation (1)-(4) are given by

Q(t)= $\frac{1}{\theta}$ (p-a) (1-$e^{-\theta t}$), 0 ≤ t ≤ t_1 -------------------------(6)

=- $\frac{a}{\theta}$ + (Q_1+$\frac{a}{\theta}$)$e^{\theta(t1-t)}$, t_1 ≤ t ≤ t_2 ------------------------(7)

= a(t_2-t), t_2 ≤ t ≤ t_3 ---------------------------------------(8)

= (p-a) (t-t_3) – Q_2, t_3 ≤ t ≤ T------------------------------(9)

From (5) and (6), we have

$Q_1 = Q(t_1) = \frac{1}{\theta}(p\text{-}a)\,(1\text{-}e^{-\theta\, t1})$

Which implies,

$e^{\theta\, t1} = [\,1\text{-}\frac{\theta Q1}{(p-a\,)}\,]^{-1}$

$t_1 = \frac{1}{\theta}\log\,[1+(\frac{\theta Q1}{(p-a)} + \frac{\theta^2 Q1^2}{(p-a)^2})]$ ------------------------(10a)

$= \frac{Q1}{(p-a)} + \frac{\theta Q1^2}{(p-a)^2}$ -------------------------------------- (10b)

(neglecting higher power of θ, (0 < θ <<1)

Again from (5) and (7), we have

$0 = Q(t_2) = -\frac{a}{\theta} + (Q_1 + \frac{a}{\theta})\, e^{\theta\,(t1-t2)}$ ---------------- (11)

$t_2 = \frac{1}{\theta}\log[(1+\frac{\theta Q1}{a})(1 + \frac{\theta Q1}{(p-a)} + \frac{\theta^2 Q1^2}{(p-a)^2})]$ (using (10)) ---
---(12)

Using the condition $Q(t_3) = -Q_2$, we have from (8)

$a(t_2\text{-}t_3) = -Q_2$

$t_3 = \frac{Q2}{a} + t_2$ ---(13)

$t_3 = \frac{Q2}{a} + \frac{1}{\theta}\log[(1+\frac{\theta Q1}{a})(1 + \frac{\theta Q1}{(p-a)} + \frac{\theta^2 Q1^2}{(p-a)^2})\,]$ --14)

From (9) and Q(T)=0, we have

$(p\text{-}a)\,(T\text{-}t_3) = Q_2$ -------------------------------------(15)

Therefore total deterioration in (0,T)

$= [(p\text{-}a)t_1\text{-}\,Q_1] + [Q_1\text{-}a(t_2\text{-}t_1)]$

$= [\frac{(p-a)}{\theta}\log\,[1 + \left(\frac{\theta Q1}{(p-a)} + \frac{\theta^2 Q1^2}{(p-a)^2}\right) - Q1\,] + [Q1\frac{a}{\theta}\log\,[1 + \frac{\theta Q1}{a}\,]$

$= \frac{\theta Q1^2 p}{2a(p-a)}$ (Neglecting higher power of θ) ---------(16)

Therefore, deterioration cost over period (0.T)

$= \frac{C3\theta Q1^2 p}{2a(p-a)}$ -- (17)

The shortage cost over the period (0,T)

$= C_2\int_{t2}^{T}(-Q(t))dt$

$= -C_2[\int_{t2}^{t3} a(t2-t)dt + \int_{t3}^{T}((p-a)(t-t3) - Q2)dt$

by (8) and (9)

$= \frac{C2Q2^2 p}{2a(p-a)}$ (by using (13) and (15) ----------------------(18)

The inventory carrying cost over cycle (0.T)

$= C_1\int_0^{t2} Q(t)dt$

$= C_1[\int_0^{t1}\frac{(p-a)}{\theta}(1-e^{-\theta t})dt + \int_{t1}^{t2}(-\frac{a}{\theta} + (Q1 + \frac{a}{\theta})\,e^{\theta\,(t1-t)})dt]$ --(19)

Now $\int_0^{t1}\frac{(p-a)}{\theta}(1-e^{-\theta t})dt$

$= \frac{(p-a)}{2}t_1^2(1 - \frac{\theta t1}{3})$ (Neglecting higher power of θ)

$= \frac{Q1^2}{2(p-a)} + \frac{\theta Q1^3}{3(p-a)^2}$ (using (10) and neglecting higher power of θ) ---(20)

$=\int_{t1}^{t2}(-\frac{a}{\theta} + (Q1 + \frac{a}{\theta})\,e^{\theta\,(t1-t)})dt$

$= \frac{a}{\theta}\,(t_1\text{-}t_2) + (Q1 + \frac{a}{\theta})\frac{1}{\theta}(1\text{-}\,e^{\theta\,(t2-t1)}$

$= \frac{Q1^2}{2a}$ (Neglecting higher power of θ) ------------ (21)

Therefore, the inventory carrying cost over the cycle (0,T)

$= C_1\,(\frac{Q1^2}{2(p-a)} + \frac{\theta Q1^3}{3(p-a)^2} + \frac{Q1^2}{2a}\,) = C_1\,(\frac{Q1^2 p}{2a(p-a)} + \frac{Q1^3}{3(p-a)2}\,\theta)$ ---(22)

Hence the total inventory cost of the system (using (17), (18) and (22))

$= TC(Q_1,Q_2)$

$= \frac{C1}{T}\,(\frac{Q1^2 p}{2a(p-a)} + \frac{Q1^3}{3(p-a)2}\,\theta) + \frac{C2Q2^2 p}{2aT(p-a)} + \frac{C3\theta Q1^2 p}{2aT(p-a)})$ -----
---(23)

From (14) and (15), we have

$Q_2 = \frac{aT(p-a)}{p} - Q_1 - \frac{\theta Q1^2(2a-p)}{2a(p-a)}$ --------------------- (24)

Therefore Using (23) and (24), the total inventory cost of the system

$=TC(Q_1) = \frac{C1}{T}\,(\frac{Q1^2 p}{2a(p-a)} + \frac{Q1^3}{3(p-a)2}\,\theta) +$

$\frac{C2(\frac{aT(p-a)}{p} - Q1 - \frac{\theta Q1^2(2a-p)}{2a(p-a)})^2 p}{2aT(p-a)} + \frac{C3\theta Q1^2 p}{2aT(p-a)}\,)$ -----(25)

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Theorem-1: The average system cost function $TC(Q_1)$ is strictly convex when $0<\theta<1$

Proof: using (25) we have,

$$\frac{dTC(Q1)}{dQ1} = \frac{C1}{T}\left(\frac{Q1p}{a(p-a)} + \frac{Q1^3\theta}{(p-a)2}\right) - \frac{C2PQ2}{aT(p-a)}$$

$$\left(1+\frac{\theta Q1(2a-p)}{a(p-a)}\right) + \frac{C3\theta Q1p}{aT(p-a)}\ ) \text{ ------------------------------(26)}$$

$$\frac{d^2TC(Q1)}{dQ1^2} = \frac{C1}{T}\left(\frac{p}{a(p-a)} + \frac{2Q1\theta}{(p-a)2}\right) + \frac{C2P}{aT(p-a)}$$

$$\left(1+\left(\frac{\theta Q1(2a-p)}{a(p-a)}\right)^2 - \left(\frac{\theta Q2(2a-p)}{a(p-a)}\right)\right) + \frac{C3\theta p}{aT(p-a)}\ ) > 0 \text{ -----(27)}$$

as $0<\theta<<1$ and $p>a$

Therefore $TC(Q_1)$ is strictly convex when $0<\theta<<1$

As $TC(Q_1)$ is strictly convex in Q_1, there exist a unique optimal stock level Q_1^* that minimizes $TC(Q_1)$. This optimal Q_1^* is the solution of the equation $\frac{dTc}{dQ1} = 0$?

We therefore find from (26) that Q_1^* is the unique root of the following equation in Q_1

$$\frac{C1}{T}\left(\frac{Q1p}{a(p-a)} + \frac{Q1^3\theta}{(p-a)2}\right) - \frac{C2PQ2}{aT(p-a)}\left(1+\frac{\theta Q1(2a-p)}{a(p-a)}\right) +$$

$$\left(\frac{C3\theta Q1p}{aT(p-a)}\right) = 0 \text{ ---(28)}$$

Where Q_2 is given by (24)

After some calculations neglecting higher power, we have

$$Q_1^* = \frac{a(p-a)C2T}{P(C1+C2)}\left[1 - \frac{C1C2T(P-a)^2 + C3p^{2(C1+C2)\theta}}{p^2(C1+C2)^2}\right] \text{ ------(29)}$$

Which is a decreasing function of θ where $0<\theta<<1$.
From (24) the optimal backlog level is given by

$$Q_2^* = \frac{a(p-a)C1T}{P(C1+C2)} + \frac{a(p-a)C2T}{P(C1+C2)}\left[\frac{C1C2T(P-a)^2 + C3p^{2(C1+C2)\theta}}{p^2(C1+C2)^2} + \right.$$

$$\left.\frac{(p-2a)C2T}{2p(C1+C2)}\right]\theta \text{ --- (30)}$$

Therefore Q_2^* is an increasing or decreasing function of θ if

$$\frac{C1C2T(P-a)^2 + C3p^{2(C1+C2)\theta}}{p^2(C1+C2)^2} + \frac{(p-2a)C2T}{2p(C1+C2)} > \text{or} < 0$$

respectively

Fuzzy Model:

The deterioration is considered as imprecise in nature and it is possible to describe it with triangular fuzzy number (symmetric). Them the deterioration is $\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$

Then the cost function is

$$TC = \frac{1}{T}\left[C_1\left\{\frac{Q1^2p}{2a(p-a)} + \frac{\tilde{\theta}\,Q1^3}{3(p-a)^2}\right\} + \frac{C2p}{2a(p-a)}\left(\frac{aT(p-a)}{p} - Q_1 - \tilde{\theta}/2\frac{(2a-p)Q1^2}{a(p-a)}\right)^2 + \frac{\tilde{\theta}\,C3pQ1^2}{2a(p-a)}\right]$$

$$= (TC_1, TC_2, TC_3)$$

Where,

$$TC_1 = \frac{1}{T}\left[C_1\left\{\frac{Q1^2P}{2a(p-a)} + \frac{\theta 1Q1^3}{3(p-a)^2}\right\} + \frac{C2p}{2a(p-a)}\left\{\frac{aT(p-a)}{p} - Q_1\right.\right.$$

$$\left.\left.- \theta_1/2\frac{(2a-p)Q1^2}{a(p-a)}\right\}^2 + \frac{\theta 1C3pQ1^2}{2a(p-a)}\right]$$

$$TC_2 = \frac{1}{T}\left[C_1\left\{\frac{Q1^2P}{2a(p-a)} + \frac{\theta 2Q1^3}{3(p-a)^2}\right\} + \frac{C2p}{2a(p-a)}\left\{\frac{aT(p-a)}{p} - Q_1\right.\right.$$

$$\left.\left.- \theta_2/2\frac{(2a-p)Q1^2}{a(p-a)}\right\}^2 + \frac{\theta 2C3pQ1^2}{2a(p-a)}\right]$$

and $$TC_3 = \frac{1}{T}\left[C_1\left\{\frac{Q1^2P}{2a(p-a)} + \frac{\theta 3Q1^3}{3(p-a)^2}\right\} + \frac{C2p}{2a(p-a)}\right.$$

$$\left.\left\{\frac{aT(p-a)}{p} - Q_1 - \theta_3/2\frac{(2a-p)Q1^2}{a(p-a)}\right\}^2 + \frac{\theta 3C3pQ1^2}{2a(p-a)}\right]$$

To defuzzify the cost function we will introduce the signed distance method and graded mean integration method. We know for any a and 0 belongs to R, the signed distance from a to 0 is $d_0(a,0) = a$. If $a<0$ the distance from a to 0 is–

$a = - d_0(a,0)$. Let ψ be the family of all fuzzy sets B is defined on R for which α – cut $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$ exists for every α belongs to [0,1]. Both $B_L(\alpha)$ and $B_U(\alpha)$ are continuous functions on α belongs to [0,1].

Then we can say for any fuzzy set B belongs to ψ we have fuzzy set $B = U\,[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$

$0\leq\alpha\leq1$

So, for fuzzy set B belongs to ψ we can define the signed distance of $\dot{B}$ to $\tilde{O}$ (y axis) as

$$d(\dot{B}, \tilde{O}) = \frac{1}{2}\int_0^1 (BL(\alpha) + BU(\alpha))\, d\alpha$$

And the graded mean integration representation of $\dot{B}$ is defined as

$$P(\dot{B}) = \frac{\frac{1}{2}\int_0^1 \alpha(BL(\alpha)+BU(\alpha))\, d\alpha}{\int_0^1 \alpha\, d\alpha}$$

For the triangular fuzzy number Ã = (a,b,c),

The α -cut of Ã is $A(\alpha) = [A_L(\alpha), A_U(\alpha)]$, for α belongs to [0,1]

Where $A_L(\alpha)=a+(b-a)\alpha$ and $A_U(\alpha) = c-(c-b)\alpha$

The signed distance of Ã to Õ (y axis) is d(Ã, Õ)= $\frac{(a+2b+c)}{4}$

The graded mean integration representation of Ã is p(Ã)=$\frac{(a+4b+c)}{6}$

*i) **Signed Distance Method:***

$$TC_S=\frac{TC1+2TC2+TC3}{4} = \frac{1}{4T}[\{C_1\{\frac{Q1^2P}{2a(p-a)}+\frac{\theta 1Q1^3}{3(p-a)^2}\} + \frac{C2p}{2a(p-a)}\{\frac{aT(p-a)}{p} - Q_1 - \theta_1/2\frac{(2a-p)Q1^2}{a(p-a)}\}^2 + \frac{\theta 1C3pQ1^2}{2a(p-a)}\}$$
$$+2\{C_1\{\frac{Q1^2P}{2a(p-a)}+\frac{\theta 2Q1^3}{3(p-a)^2}\} + \frac{C2p}{2a(p-a)}\{\frac{aT(p-a)}{p} - Q_1 - \theta_2/2\frac{(2a-p)Q1^2}{a(p-a)}\}^2 + \frac{\theta 2C3pQ1^2}{2a(p-a)}\}+\{C_1\{\frac{Q1^2P}{2a(p-a)}+\frac{\theta 3Q1^3}{3(p-a)^2}\} + \frac{C2p}{2a(p-a)}\{\frac{aT(p-a)}{p} - Q_1 - \theta_3/2\frac{(2a-p)Q1^2}{a(p-a)}\}^2 + \frac{\theta 3C3pQ1^2}{2a(p-a)}\}]$$

*ii) **Graded Mean Integration Method:***

$$TC_G=\frac{TC1+4TC2+TC3}{6} = \frac{1}{6T}[\{C_1\{\frac{Q1^2P}{2a(p-a)}+\frac{\theta 1Q1^3}{3(p-a)^2}\} + \frac{C2p}{2a(p-a)}\{\frac{aT(p-a)}{p} - Q_1 - \theta_1/2\frac{(2a-p)Q1^2}{a(p-a)}\}^2 + \frac{\theta 1C3pQ1^2}{2a(p-a)}\} +$$
$$4\{C_1\{\frac{Q1^2P}{2a(p-a)}+\frac{\theta 2Q1^3}{3(p-a)^2}\} + \frac{C2p}{2a(p-a)}\{\frac{aT(p-a)}{p} - Q_1 - \theta_2/2\frac{(2a-p)Q1^2}{a(p-a)}\}^2 + \frac{\theta 2C3pQ1^2}{2a(p-a)}\}+\{C_1\{\frac{Q1^2P}{2a(p-a)}+\frac{\theta 3Q1^3}{3(p-a)^2}\} + \frac{C2p}{2a(p-a)}\{\frac{aT(p-a)}{p} - Q_1 - \theta_3/2\frac{(2a-p)Q1^2}{a(p-a)}\}^2 + \frac{\theta 3C3pQ1^2}{2a(p-a)}\}]$$

IV. NUMERICAL EXAMPLES

Here we have calculated optimal stock level Q_1^*, optimal backlog level Q_2^*, the minimum average system cost TC^* for given values of production cycle length T and other parameters by considering example.

Example:

We consider the following numerical values of the parameters in appropriate units to analyze the model

$\theta = 0.0004$, $C_1=4$, $C_2=20$, $C_3=40$, p=20, a=8 and T= 80 in appropriate unit

We obtain for crisp model $Q_1^*= 319.982$, $Q_2^*= 64.871$, $TC^*= 646.514$

For fuzzy model, we consider the following numerical values of the parameters in appropriate units to analyze the fuzzy model ~

θ = (0.0002,0.0004,0.0006)

we obtain

Foe signed distance method $Q_{1S}^* = 319.98$, $Q_{2S}^*= 64.87$, $TC_S^*= 646.516$

For graded mean integration method $Q_{1G}^* = 295.06$, $Q_{2G}^*= 89.66$, $TC_G^*= 596.02$

V. SENSITIVITY ANALYSIS

The sensitivity analysis is performed by changing the value of each of the parameters C_1, C_2, C_3, p, a and T taking one parameter at each time and keeping the remaining parameters unchanged. We now study sensitivity of the optimal solution to changes in the values of different parameters associated with the model.

Table 1: Sensitivity on 'C_1'

Change value	Signed Distance Method			Graded Mean Integration Method		
C_1	Q_{1C}	Q_{2C}	TC_c	Q_{1G}	Q_{2G}	TC_G
2	349.42	35.59	354.49	333.93	50.10	338.73
3	334.05	50.88	506.93	313.28	71.53	475.31
5	307.06	77.72	774.81	278.86	105.79	703.43
6	295.15	89.57	893.13	264.36	120.23	799.65

Table 2: Sensitivity on 'C_2'

Change value	Signed Distance Method			Graded Mean Integration Method		
C_2	Q_{1C}	Q_{2C}	TC_c	Q_{1G}	Q_{2G}	TC_G
14	298.38	86.36	602.71	268.24	116.35	541.69
17	310.72	74.09	627.74	283.39	101.28	572.38
23	327.20	57.09	661.13	304.33	80.43	614.81

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26	332.98	51.95	672.84	311.87	72.94	630.09

Table 3: Sensitivity on 'C_3'

Change value	**Signed Distance Method**			**Graded Mean Integration Method**		
C_3	Q_{1c}	Q_{2c}	TC_c	Q_{1G}	Q_{2G}	TC_G
30	320.0	64.82	645.98	295.13	89.59	595.57
35	320.01	64.84	646.25	295.09	89.63	595.80
45	319.95	64.90	646.78	295.03	89.70	596.25
50	319.93	64.92	647.05	294.99	89.73	596.48

Table 4: Sensitivity on 'p'

Change value	**Signed Distance Method**			**Graded Mean Integration Method**		
E	Q_{1c}	Q_{2c}	TC_c	Q_{1G}	Q_{2G}	TC_G
16	266.19	53.81	537.08	245.58	74.42	495.37
18	296.06	59.94	597.77	273.06	82.87	551.20
22	339.58	68.93	686.54	313.08	95.24	632.80
24	355.92	72.33	719.98	328.10	99.91	663.52

Table 5: Sensitivity on 'a'

Change value	**Signed Distance Method**			**Graded Mean Integration Method**		
A	Q_{1c}	Q_{2c}	TC_c	Q_{1G}	Q_{2G}	TC_G
6	280.44	57.06	567.66	258.47	78.80	523.05
7	303.56	61.65	613.88	279.86	85.17	565.79
9	329.70	66.74	665.65	304.10	92.28	613.81
10	332.74	67.26	671.35	306.97	93.02	619.21

Table 6: Sensitivity on 'T'

Change value	**Signed Distance Method**			**Graded Mean Integration Method**		
T	Q_{1c}	Q_{2c}	TC_c	Q_{1G}	Q_{2G}	TC_G
70	279.96	56.69	565.22	258.18	78.37	521.14
75	299.97	60.78	605.85	276.62	84.01	558.57
85	339.99	68.97	687.22	313.50	95.32	633.51
90	360.01	73.07	727.95	331.94	100.98	671.01

Observation:

The following are noted on the basis of the sensitivity analysis-

i) From Table 1, 2 and 3 it is observed that, increase (or decrease) of the various cost C_1, C_2, and C_3, the total inventory cost (for the two models) also increases (or decrease). Also, we observed that TC_c and TC_G are highly sensitive to change in 'C_1, C_2', but insensitive to change in C_3.

ii) As the production rate e and cycle time T increases (or decrease) we observed that the total cost TC_c and TC_G increases (or decrease). Here also TC_c and TC_G are highly sensitive due to change in 'e' and 'T'. (From table 4 and 6).

iii) Also, we observed from table 5 the total cost TC_c and TC_G (for both the models) increases (or decreases) as the demand per unit time 'a' increases (or decreases). For both the models TC_c and TC_G are moderately sensitive to changes in the value of the parameter a.

VI. CONCLUSION

In the present chapter, we have dealt with a continuous production control inventory model for deteriorating items with shortage where we have described deterioration rate as triangular fuzzy number (symmetric). Signed distance method and graded mean integration method are used as the method of defuzzification to find the estimate of total cost per unit time in the fuzzy sense. It is assumed that the demand and production rates are constant and the distribution of the time to deterioration of an item follows the

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exponential distribution. This model is applicable for food items, drugs, pharmaceuticals. Here we have studied the structural properties of this inventory system. I have tried to compare Signed distance method with the graded mean integration method and have seen that the total cost obtained by graded mean integration method is less than that of obtained by signed distance method. From the sensitivity analysis it is observed that the total cost of both the model increase as the cost associated with the model increase.

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