

Numerical Solution of Nonlinear Equations in Maple

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ABSTRACT

The main purpose of this paper is to obtain the real roots of an expression using the Numerical method, bisection method, Newton's method and secant method. Root analysis is calculated using specific, precise starting points and numerical methods and is represented by Maple. In this research, we used Maple software to analyze the roots of nonlinear equations by special methods, and by showing geometric diagrams, we examined the relevant examples. In this process, the Newton-Raphson method, the algorithm for root access, is fully illustrated by Maple. Also, the secant method and the bisection method were demonstrated by Maple by solving examples and drawing graphs related to each method.

Keywords- Numerical method, Bisection method, Newton's method and Secant method.

I. INTRODUCTION

Maple provides flexible tools for numerical root finding using algorithms, such as Newton's method and the bisection method. The field of numerical analysis focuses on algorithms that use numerical approximations for mathematical analysis problems. Maple provides algorithms for solving nonlinear and root equations through a variety of numerical methods [8]. One of the most important topics in numerical analysis is the numerical solution of nonlinear equations, so the need for efficient methods to solve such equations is felt more than ever. Since most nonlinear equations can not be solved directly by analytical methods, approximation-based methods must be used to solve them. For this purpose, there are very effective methods that can be used with the necessary accuracy [10]. In this research, we discuss some very important methods of numerically solving nonlinear equations. Also, in this research, we tried to show nonlinear equations by analysis, analysis of nonlinear equations, graphs and some computational capabilities of Maple software, including numerical solution and rooting of equations [9]. In Maple software, many computational problems are provided with accurate and explicit answers. Maple software also has many capabilities, including converting outputs to MATLAB code and LaTeX commands.

II. MATERIALS AND METHODS

Because of these tasks, the calculation of roots is accurately demonstrated by Maple using the following methods.

A. Bisection Method

The first numerical method we study is the method of halving the distance [3]. In mathematics, the halving method is a method of solving equations that is used for continuous functions. This method involves repeating halves of the distances defined by the values, and the distances are reduced to the desired root. This method for solving the equation is a very simple and powerful method that is used to solve all kinds of equations, but it is a relatively slow method. To start the method, the values of the function for a series of points are calculated until two points a and b are obtained, the corresponding values of which are different. Assuming that the function is continuous, a square root should be placed between a and b [1]. Suppose $f(x)$ is continuous at the given distance $[a, b]$ and holds on $f(a)f(b) < 0$. The function $f(x)$ must have at least one root in $[a, b]$. Usually $[a, b]$ is chosen so that it contains only one square root, but the following algorithm, for the halving method due to $f(a)f(b) < 0$, always has the square root in $[a, b]$ It is convergent [2]. Finding a root is as simple as halving the interval $[a, b]$ and keeping the half for which $f(x)$ changes sign. This method is known as the bisection method, and it is guaranteed to converge to a root, which is denoted by in this case [4]. To numerically estimate the roots of an algebraic function, the Bisection command uses a basic binary search technique. Given the expression f and the expression [7].

Examples:

➤ $f := x^3 - 7x^2 + 14x - 6$

$f := x^3 - 7x^2 + 14x - 6$

➤ $\text{Bisection}(f, x = [2.6, 3.2], \text{tolerance} = 10^{-2})$

2.993750000

➤ $\text{Bisection}(f, x = [2.7, 3.1], \text{tolerance} = 10^{-2}, \text{output} = \text{sequence})$

[2.7, 3.1], [2.900000000, 3.1], [3.000000000, 3.1], [3.000000000, 3.050000000], 3.025000000

➤ $\text{Bisection}(f, x = [2.7, 3.2], \text{tolerance} = 10^{-2}, \text{stoppingcriterion} = \text{absolute})$

3.004687500

➤ $\text{Bisection}(f, x = [2.7, 3.2], \text{output} = \text{animation}, \text{tolerance} = 10^{-3}, \text{stoppingcriterion} = \text{function_value})$

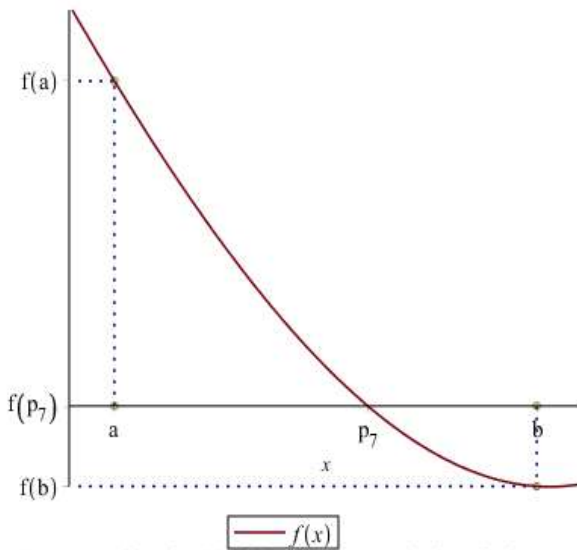


figure 1 - 7 iteration(s) of the bisection method applied to $f(x) = x^3 - 7x^2 + 14x - 6$ with initial points $a = 2.7$ and $b = 3.2$.

`Bisection(f, x = [2.95, 3.05], output = plot, tolerance = 10-3, maxiterations = 10, stoppingcriterion = relative)`

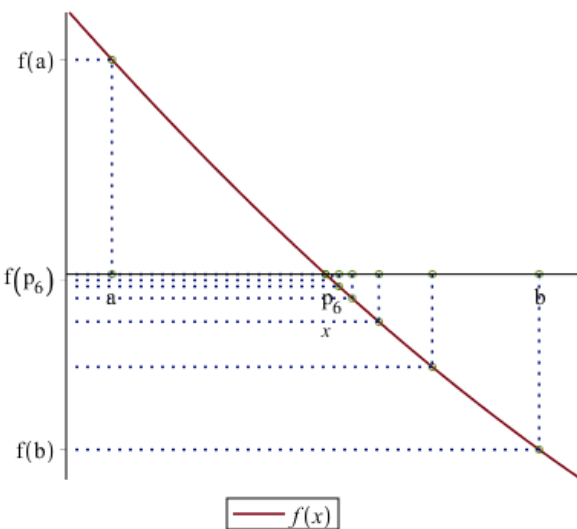


figure 2 - 6 iteration(s) of the bisection method applied to $f(x) = x^3 - 7x^2 + 14x - 6$ with initial points $a = 2.95$ and $b = 3.05$.

B. Newton-Raphson Method

Newton's method, also known as Newton-Raphson algorithm, is known as one of the most efficient numerical methods in mathematics for approximating the square root of nonlinear equations on the field of real numbers. Calculating the square root of a function using the Newton-Raphson method is one of the most accurate and fastest ways to find the square root of equations or determine the square root of a function. There are different ways to obtain the formula of this method [5].

Suppose we want to find the approximate solution of the equation $f(x) = 0$. Also call the initial approximation of the answer of equation x_0 . This initial

approximation may not be appropriate, but it is normal because it is a quick initial guess. So we have to find a better approximation. To do this, consider the line tangent to $f(x)$ at the point x_0 . When used with the Newton command, it numerically approximates the roots of an algebraic function, f , using the Newton-Raphson method, which is the most widely used method in computer science. The Newton command computes a sequence of numbers given an expression f and an initial approximate $p_k, k = 0..n$, of approximations to a root of f , where n is the number of times a stopping criterion is met in a given amount of time. When the functions are sufficiently well-behaved and the initial approximations are sufficiently good, the convergence of p_k toward the exact root is quadratic. When the Newton command is used, it is a shortcut for invoking the Roots command with the method=newton parameter. [6].

- Newton's method will fail if $\frac{\partial}{\partial x} f(p_{k-1}) = 0$.

$$\frac{d}{dx} f(p_{k-1}) = 0$$

Examples:

`> f := ex + 2-x + 2 cos(x) - 6`

`f := ex + 2-x + 2 cos(x) - 6`

`> Newton(f, x = 3.0, tolerance = 10-2)`

1.829383705

`> Newton(f, x = 3.0, tolerance = 10-2, output = sequence)`

3.0, 2.379684188, 1.997823802, 1.850021156, 1.829734165, 1.829383705

`> Newton(f, x = 8, output = plot, stoppingcriterion = function_value)`

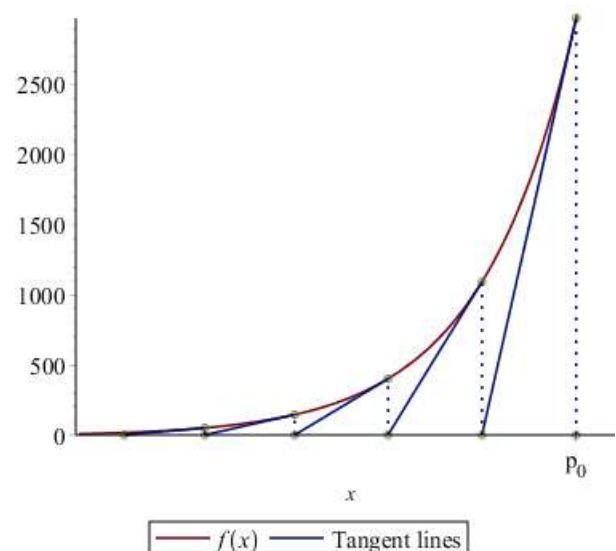


figure 3 - 5 iterations of Newton's method applied to $x) = e^x + 2^{-x} + 2 \cos(x) - 6$ with initial point $p_0 = 8$. The stopping criterion is not met.

`> Newton(f, x = 1.3, output = animation, stoppingcriterion = absolute)`

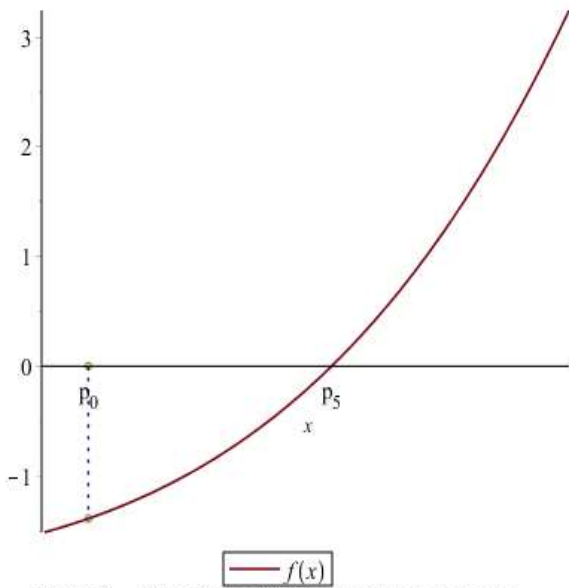


figure 4 - 5 iterations of Newton's method applied to $f(x) = e^x + 2^{-x} + 2 \cos(x) - 6$ with initial point $p_0 = 1.3$.

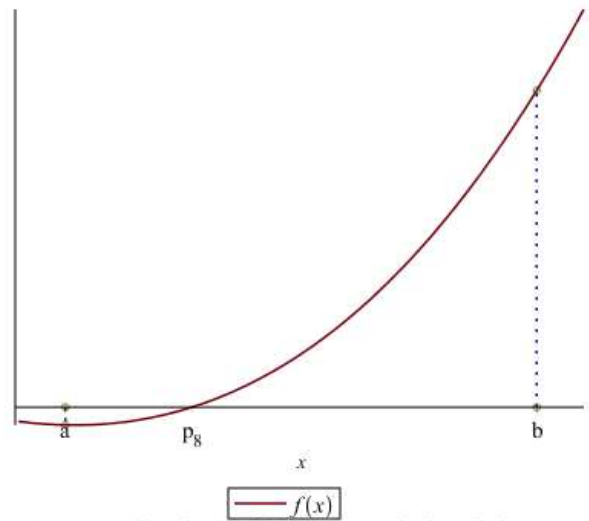


figure5 - 8 iteration(s) of the secant method applied to $f(x) = x^3 - 7x^2 + 14x - 6$ with initial points $a = 3.2$ and $b = 4.0$.

>Secant($f, x = [2.9, 3.1]$, tolerance = 10^{-3} , output = plot)

C. Secant Method

When used in conjunction with the Secant command, the roots of an algebraic function, f , are numerically approximated using a technique similar to Newton's method, but without the requirement of evaluating the derivative of f . Given an expression f and an initial approximate a , the Secant command computes a sequence $p_k, k = 0..n$, of approximations to a root of f , where n is the number of iterations taken to reach a stopping criterion. It is possible to call the Roots command with the method=secant option without having to type in the full path to the command. However, the secant method has the limitation that it may diverge when the initial approximates a and b do not come close enough to the root.

Examples:

> $f := x^3 - 7x^2 + 14x - 6$

$f := x^3 - 7x^2 + 14x - 6$

> Secant($f, x = [1.7, 3.2]$, tolerance = 10^{-2})

2.997468286

>

Secant($f, x = [1.7, 3.2]$, tolerance = 10^{-2} , output = sequence)

1.7, 3.2, 3.135260116, 2.741232363, 3.054880675, 3.018866241, 2.997468286

>

Secant($f, x = [1.7, 3.2]$, tolerance = 10^{-2} , stoppingcriterion = absolute)

3.000099582

>

Secant($f, x = [3.2, 4.0]$, output = animation, stoppingcriterion = function_value)

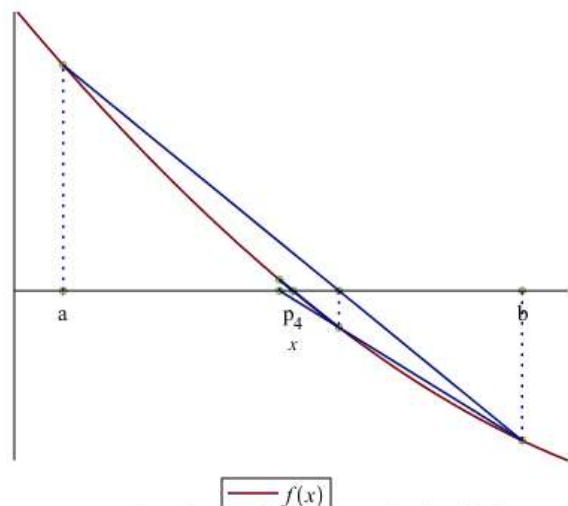


figure 6 - 4 iteration(s) of the secant method applied to $f(x) = x^3 - 7x^2 + 14x - 6$ with initial points $a = 2.9$ and $b = 3.1$.

III. CONCLUSION

The results show that Maple software has good computational capabilities, and can be very useful for analyzing methods for obtaining the root of nonlinear equations of a software. In this research, we were able to study several special methods related to obtaining the roots of nonlinear equations separately and showed each of these methods by solving questions and drawing geometric graphs.

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